

# Converse, Inverse, Contrapositive, and Biconditionals

Unit 2: Logic and Reasoning • Day 3 of 11

## At a Glance

**Pacing:** ~45 min task + curated gallery walk • ~45 min consolidation (guided notes + CYU begins)

**Materials:** Boards and markers

**Nav instrument rows active:** Rows 1–2 spiral • Row 3 (Conditional Statements) full activation today — all four forms, biconditional, inverse error

## OBJECTIVES

By the end of this lesson students can:

- Write the converse, inverse, and contrapositive of a conditional statement.
- Determine the truth value of each of the four forms and give counterexamples for false ones.
- Explain why the contrapositive is equivalent to the original using a Venn diagram.
- Identify biconditional statements and write them in if-and-only-if form.
- Name and explain the inverse error.

## TASK

### Task — What Happens When You Flip or Negate? (~30 min)

Write only the following on the board. Give no further instructions.

#### Starting statement

**"If a number is divisible by 4, then it is divisible by 2."**

Write as many variations of this statement as you can — by changing the order of the parts, by negating parts, or both. For each variation: is it true or false? How do you know? Find a counterexample if it is false. When you have all the variations you can find, look at your results. Is there a pattern to which ones are true and which are false?

**Extension ladder:** Once groups find all four forms, try the same process with: (1) 'If a figure is a square, then it has four equal sides.' (2) 'If it is raining, then the ground is wet.' Does the same pattern hold? Then, separately: (3) 'If it is February 12th, then it is President Lincoln's birthday.' Does the same pattern hold for this one? What is different about it? Can you draw a Venn diagram for it?

**What to look for during circulation:** Groups that found only 2–3 variations — prompt: 'Are there others?' Groups that noticed the pairing pattern before you named it. Any group working on the Lincoln extension — watch whether they notice something is different about it (both directions are true).

## CURATED GALLERY WALK (~8 MIN)

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Identify boards during circulation. Move from less complete to more complete:

- A group that found 3 variations — ask: what is the fourth?
- A group with all four forms and truth values — ask them to explain one false one.
- A group that noticed the pairing pattern — let them present it.

Key question to pose: ‘Two groups noticed the same pattern in truth values. Can anyone explain why that pattern holds?’ Hold the answer for consolidation.

## WHAT TO WATCH FOR

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### ⚠ Watch for

**Most common error:** Groups who test only one statement (e.g. divisible by 6) and assume the pattern holds universally. Push them to test a case where the original is true and the converse could go either way.

**Lincoln extension:** This is the biconditional preview. If a group reaches it and notices both directions are true, do not explain it yet — note the board for the gallery walk and let it drive the biconditional moment in consolidation.

**Payoff from 2.2a:** The watched pot question from last class — students can now answer it. The inference ‘an unwatched pot must boil’ is the inverse error. Name this during consolidation.

## CONSOLIDATION

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Build the four-forms table together using students’ work from the boards. For each row: introduce the name, ask students if they can guess what it describes, give the definition and symbolic form, then ask for the example from the divisibility task.

After the table, pause: ‘Why does the contrapositive always match the conditional? Is this a coincidence?’ Let students speculate, then draw the Venn diagram together.

### The Venn diagram explanation — draw this with the class

*The  $p$ -circle sits entirely inside the  $q$ -circle. If something is outside  $q$ , it must be outside  $p$  — that is the contrapositive. If something is inside  $q$ , it may or may not be inside  $p$  — that is why the converse is not guaranteed. The inverse and converse fail for the same reason: they reverse the nesting direction.*

*Students who see this do not need to memorize four separate truth patterns. They need to understand one diagram.*

Close with two payoffs:

**Payoff 1 — Biconditional**

When both  $p \rightarrow q$  and  $q \rightarrow p$  are true, the two circles occupy exactly the same space.  
Symbolic form:  $p \leftrightarrow q$ . 'p if and only if q.'

The Lincoln birthday extension is the example: both directions are true, the Venn circles completely overlap. Ask: can they think of another example? Then name the most important one: all definitions are biconditional. A definition must classify correctly in both directions — it includes everything that fits and excludes everything that does not.

**Payoff 2 — Return to the watched pot (the inverse error)**

Return to the statement from Lesson 2.2a: 'If it doesn't rain, we will go outside.' Students inferred: if it rains, we stay inside. They now have language for what went wrong.

That inference assumes the inverse follows from the conditional. It does not — the inverse and conditional are not equivalent. This reasoning error is called the inverse error. Its formal name is denying the antecedent — assuming that because the hypothesis is false, the conclusion must be false. Students will see this named again in Lesson 2.4.

**TEACHER REFERENCE — COMPLETED FOUR-FORMS TABLE**

| Name           | Symbolic                    | In Words (divisibility example)                 | True?          | Equivalent to... |
|----------------|-----------------------------|-------------------------------------------------|----------------|------------------|
| Conditional    | $p \rightarrow q$           | If divisible by 4, then divisible by 2.         | TRUE           | Contrapositive   |
| Converse       | $q \rightarrow p$           | If divisible by 2, then divisible by 4.         | FALSE (e.g. 6) | Inverse          |
| Inverse        | $\neg p \rightarrow \neg q$ | If not divisible by 4, then not divisible by 2. | FALSE (e.g. 6) | Converse         |
| Contrapositive | $\neg q \rightarrow \neg p$ | If not divisible by 2, then not divisible by 4. | TRUE           | Conditional      |

**DEFINITIONS AND KEY POINTS**

**Four forms — names and symbolic forms to confirm students have correctly**

**Conditional:**  $p \rightarrow q$  (original)

**Converse:**  $q \rightarrow p$  (swap hypothesis and conclusion)

**Inverse:**  $\neg p \rightarrow \neg q$  (negate both)

**Contrapositive:**  $\neg q \rightarrow \neg p$  (swap and negate both)

**Equivalence pairs:** Conditional  $\leftrightarrow$  Contrapositive (always same truth value) • Converse  $\leftrightarrow$  Inverse (always same truth value)

### Biconditional

$p \leftrightarrow q$  ('p if and only if q,' or 'p iff q'). True when both the conditional and its converse are true.

Key point: all definitions are biconditional. A precise definition must work in both directions.

### Inverse error (denying the antecedent)

The error of assuming the inverse is true because the conditional is true. Formally:  $p \rightarrow q$  does not imply  $\neg p \rightarrow \neg q$ . Preview name for Lesson 2.4: denying the antecedent.

## STUDENTS MAKE NOTES ON

### Students make notes on:

- The four-forms table — name, symbolic form, and one worked example for each (the divisibility example from the task)
- Venn diagram showing why the equivalence pairs hold — students draw this themselves
- Biconditional: definition, symbolic form, one example, and the note that all definitions are biconditional
- The inverse error: what it is, one example, and its formal name (denying the antecedent)

## CONNECTION POINTS

### Draws from

- Lesson 2.2a: the conditional statement is the starting object; the watched pot question is resolved here
- Unit 1: definitions — the biconditional note connects back to the definition work in Lesson 1.1

### Feeds into

- Mid-unit quiz: covers rows 1–3; contrapositive and biconditional are assessed here
- Lesson 2.4: the inverse error gets its formal name (denying the antecedent) and joins the fallacy table
- Unit 3+: the equivalence of conditional and contrapositive is used constantly in geometric proof