

Dilation and Scale Factor

Unit 6: Similarity • Day 2 of unit

At a Glance

Pacing: Pentagon task: boards (~30 min) • Preselected group presentations (~10 min) • Group note-making at boards (~15 min) • Groups share notes + teacher synthesizes (~10 min) • Distribute student note sheets + CYU (~15 min or homework)

Materials: Projected pentagon image

Nav instrument rows active: Row 1 (Similarity and Scale Factor) — all three levels activate today

Key structural difference from 6.1: No physical apparatus. Students reason from a diagram. The formalization earned from 6.1 happens here.

Teacher decision: CYU 6.2 can be done in class (if group note-making is brisk) or assigned as homework. The problems on perimeter and area scaling are intentionally left open as previews of Lesson 6.6.

Pentagon task: One pair of corresponding sides and the distances from the light source to two corresponding vertices are given. Scale factor = 2.5. Display the image — do not label the scale factor.

OBJECTIVES

By the end of this lesson students can:

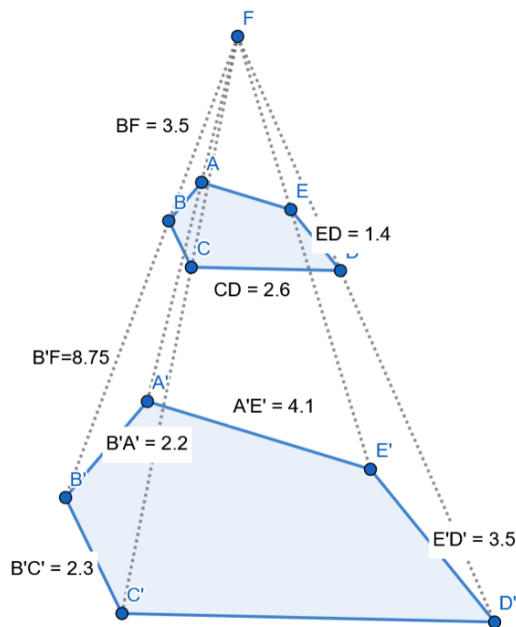
- Define dilation precisely: a transformation that scales from a fixed center point, moving every point along its ray from the center by the scale factor.
- Define scale factor and compute it in both directions (image/original and original/image).
- Write and interpret similarity statements, including correct correspondence order.
- Use a scale factor to find missing side lengths in similar figures.
- Explain that corresponding angles are preserved under dilation and corresponding sides are proportional.
- Articulate informally why perimeter scales by k and area appears to scale by k^2 (if reached in the task).

 **Narrative Arc** — Open class — the bridge from 6.1

“Yesterday you worked with flashlights and shadows. You measured, computed ratios, and noticed what stayed the same. Today the flashlight is gone. What remains is the structure. We are going to use what you found yesterday to answer questions about a figure you have never seen before.”

BOARD TASK — A SHADOW WITHOUT THE FLASHLIGHT (≈30 MIN)

Project the pentagon image on the Promethean board. Do not label the scale factor. Groups work at boards.



Assume all lengths given are in feet.

Find:

- AB
- AE
- BC
- $C'D'$

Task prompt — give verbally or display on board

Prompt

‘This pentagon is a shadow of the other. Both pentagons are on parallel planes. You are given some information but not everything. Your job is to find as many missing side lengths as you can, and to explain how you know.’

What to watch for during circulation

Using vertex distances to find scale factor: $L2/L1 = 2.5$. This connects directly to 6.1. Celebrate it explicitly when you see it.

Using corresponding side ratio: $3.5/1.4 = 2.5$. Both routes give the same scale factor.

Correspondence errors: students who compute ratios with non-corresponding sides. Ask: ‘How do you know those sides correspond? Which vertex connects to which?’

Scale factor direction confusion: students who find 0.4 (the reciprocal) instead of 2.5. Ask: ‘Is your scale factor going from smaller to larger, or larger to smaller?’

Extension prompts — give one at a time to groups who finish early

- What is the scale factor from the smaller pentagon to the larger?
- What is the scale factor from the larger to the smaller?
- If I placed another plane 5 feet from the light source, how long would each side be on that plane?
- Can you spot any similar triangles in the diagram? How many pairs are there?

- How do you think the perimeter changes as the figure moves farther from the light?
- How do you think the area changes?
- If the area of the smaller pentagon were 3 square units, what would you predict the area of the larger one to be?
- Can you test your area conjecture using a shape whose area you already know how to calculate?

These questions are intentionally unresolved today. They plant seeds for Lesson 6.6. If a group reaches the area question, listen carefully to their conjecture. It will be important to reference in 6.6.

PRESELECTED PRESENTATIONS (≈10 MIN)

Before group note-making, select three groups to present. Choose in advance as you circulate:

Three groups to preselect

Group 1: one that clearly identified the scale factor (from either vertex distances or side ratios) and used it to find missing lengths.

Group 2: one that used vertex distances specifically to justify proportionality — the 6.1 connection.

Group 3: one that made a conjecture about perimeter or area scaling (if any group reached it).

How to run the presentations

Do not correct immediately. Press on reasoning, then let other students question and refine:

- ‘Why are you allowed to use that ratio?’
- ‘How do you know those sides correspond?’
- ‘What assumptions are you making?’

After Group 1 presents: write the similarity statement on the board yourself — pentagon $ABCDE \sim$ pentagon $A'B'C'D'E'$. Explicitly model the order and the symbol. Say: ‘The order of letters encodes the correspondence. A corresponds to A’, B to B’, and so on. Writing it in a different order makes a different claim.’

GROUP NOTE-MAKING AT THE BOARDS (≈15 MIN)

After presentations, groups create their own notes at the boards. Give them these six prompts on the board:

Six note-making prompts — write on the board

1. What is a dilation?
2. What stays the same under a dilation?
3. What changes under a dilation?
4. What is a scale factor, and how is it used?
5. One worked example.
6. Space for an example of your choosing.

What to listen for as groups build their notes

On dilation: do they name the center? Do they connect it to the flashlight from 6.1? Push: 'Where is the center in the pentagon diagram?'

On what stays the same: angles and ratios of corresponding lengths. Push if students say 'the shape' — ask them to be precise.

On scale factor: do they state both directions? Do they connect k to L_2/L_1 as well as to image/original side lengths?

On examples: do they include a worked example that shows both finding a missing side and writing a similarity statement?

TEACHER SYNTHESIS — SHARE AND FORMALIZE (≈10 MIN)

Groups share their notes. After each group, refine toward the precise formulations below. These are what you want on every student's note sheet.

Term	Definition — what to surface in consolidation
Dilation	A transformation that scales a figure from a fixed point (the center of dilation). Every point moves along the ray from the center through that point, at a distance multiplied by the scale factor.
Center of dilation	The fixed point from which all scaling occurs. In the 6.1 experiment: the flashlight. In coordinate geometry: an explicit point, often $(0, 0)$ or a given coordinate.
Scale factor	The constant ratio between corresponding lengths in the image and the original: $k = \text{image length} \div \text{original length}$. Also equal to L_2 / L_1 (distance from center to image vertex $\div$ distance from center to original vertex).
Similar figures	Two figures are similar ($\sim$) if one can be obtained from the other by a dilation (possibly followed by a rigid motion). Formally: corresponding angles are equal AND corresponding sides are proportional.
Similarity statement	A statement of the form $\triangle ABC \sim \triangle DEF$. Like a congruence statement, order encodes correspondence: $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$. Writing the order incorrectly makes a false claim.
Scale factor direction	From A to B: $k = \text{image/original}$. From B to A: $k = \text{original/image} = 1/k$. Both are valid scale factors; direction matters for calculations.

Two things to name explicitly that students often miss

Similarity vs. congruence: congruence is a special case of similarity where $k = 1$. Two congruent figures are similar. Two similar figures are not necessarily congruent. Similarity is the broader relationship.

The 6.1 connection to close: yesterday's equilateral triangle shadow. The scalene triangle and the equilateral triangle produced the same shadow because shadows are projections, not dilations, when the figure and projection plane are not parallel. A dilation requires parallel planes (or a specific geometric setup). Name this explicitly now that students have the vocabulary.

Perimeter and area — if groups reached it

Perimeter: if k is the scale factor, then the perimeter of the image = $k \times$ perimeter of the original. Each side scales by k , so the sum does too.

Area: area appears to scale by k^2 . Do not prove this today. Accept conjectures and tell students this will be the focus of Lesson 6.6.

DISTRIBUTE STUDENT NOTE SHEETS

Once group note-making and sharing is complete, distribute the pre-made student note sheets. Students transfer their board notes to the sheet. The sheet has two sections: Dilation (definition + picture placeholder) and Scale Factor (definition + picture placeholder + two examples, one partially filled).

Teacher sample — project or display when distributing

Dilation: a transformation that scales a figure from a fixed point (the center). Corresponding angles remain equal. Corresponding side lengths are proportional.

Scale factor: the constant ratio between corresponding lengths (image $\div$ original). Also equal to distance from center to image vertex $\div$ distance from center to original vertex.

Example: A figure is dilated by a factor of 3. A side of length 4 becomes 12. An angle of 65° remains 65° .

STUDENTS MAKE NOTES ON

Students make notes on:

- Dilation: definition including the center, what stays the same (angles), what changes (lengths)
- Scale factor: definition, how to compute it (image/original = L_2/L_1), both directions, what it means when $k < 1$ vs. $k > 1$
- Similarity statement: notation, correspondence order, the symbol $\sim$
- Worked example: finding missing sides in a similar figure using scale factor
- Optional (if reached): perimeter scales by k , area appears to scale by k^2 — flag as to be proven in Lesson 6.6

 Narrative Arc — End of consolidation — transition to Lesson 6.3

“We now have a precise definition: two figures are similar if corresponding angles are equal and corresponding sides are proportional. That definition requires six conditions: three angle pairs and three side ratios. But in Unit 3 we discovered that for congruent triangles, we could often prove all six conditions from just two or three pieces of information. The question for Lesson 6.3: can we do the same thing for similar triangles? And how do we find out?”

**Connection
Points**

- Draws from Lesson 6.1: scale factor $k = L2/L1$ is directly from the shadow experiment; the center of dilation is the flashlight
- Draws from Unit 3: similarity statement correspondence order mirrors congruence statement correspondence from CPCTC
- Draws from Unit 1: the concept of ratio and proportion from Lesson 1.2 is the arithmetic engine of scale factor
- Feeds into Lesson 6.3: the six-conditions definition of similarity (3 angles + 3 proportional sides) sets up the search for shortcuts exactly as in Unit 3
- Feeds into Lesson 6.6: perimeter scales by k , area by k^2 — conjectures planted today are proven in that lesson