

What Makes an Argument Good?

Unit 2: Logic and Reasoning • Day 5 of 11 • Valid, Sound, Invalid — Argument Sorting Task

At a Glance

Pacing: Return quiz + nav update (5 min) • Task 1 (15 min) • Task 2 + gallery walk (35 min) • Consolidation notes + CYU begins (35 min)

Materials: Boards and markers • Nine-argument table printed or projected for Task 2

Nav instrument rows active: Rows 1–3 spiral • Row 4 (Evaluating Arguments) activates today

OBJECTIVES

By the end of Days 1 and 2 students can:

- Distinguish between valid, sound, and invalid arguments.
- Recognize and name the three fallacies: affirming the consequent, denying the antecedent, undistributed middle.
- Explain why each fallacy fails using a Venn diagram.
- Connect each fallacy and valid form to the four conditional forms from Lesson 2.2b.

TASKS

Task 1 — Always, Sometimes, or Never? (≈15 min)

Source: NRICH, University of Cambridge — rich.maths.org/problems/always-sometimes-or-never-number

Groups sort mathematical statements into always true, sometimes true, or never true. For ‘sometimes’ statements, they must specify when.

The sum of three numbers is odd	If you add 1 to an odd number you get an even number
Multiples of 5 end in a 5	If you add two odd numbers you get an odd number
If you add a multiple of 10 to a multiple of 5 the answer is a multiple of 5	
When you multiply two numbers you will get a bigger number	If you add a number to 5 your answer will be bigger than 5
A square number has an even number of factors	The sum of three consecutive numbers is divisible by 3
Dividing a whole number by a half makes it twice as big	

Transition to Task 2: Write on the board: Sound. Valid. Invalid. Say: ‘A sound argument is one where everything works — the logic and the facts. That’s all I’ll tell you.’

Task 2 — Is This a Good Argument? (≈25 min)

Groups create three columns on their boards: Sound, Valid, Invalid. They sort arguments A–I and write a reason for each placement.

ID	Premises and Conclusion	Your Placement	Written Reason
A	P1: All cats are mammals. P2: Felix is a cat. $\therefore$ Felix is a mammal.		
B	P1: All dogs are animals. P2: All cats are animals. $\therefore$ All cats are dogs.		
C	P1: If it rains, the ground gets wet. P2: It is raining. $\therefore$ The ground is wet.		
D	P1: If I eat broccoli, I'll be healthy. P2: I am healthy. $\therefore$ I must have eaten broccoli.		
E	P1: All humans are mortal. P2: Socrates is mortal. $\therefore$ Socrates is a human.		
F	P1: All unicorns have horns. P2: Steve is a unicorn. $\therefore$ Steve has a horn.		
G	P1: If aliens exist, then Area 51 is hiding them. P2: Aliens exist. $\therefore$ Area 51 is hiding them.		
H	P1: If the moon is made of cheese, then it tastes delicious. P2: The moon is made of cheese. $\therefore$ It tastes delicious.		
I	P1: All Martians wear hats. P2: Fred is a Martian. $\therefore$ Fred wears a hat.		

Watch for

F and H (unicorn/moon-cheese): Groups who put these in Invalid because the premises are absurd. This is the valid/sound confusion — the most important distinction to surface. These arguments are valid. The logic holds; the premises just happen to be false.

B, D, E: Groups who correctly identify these as invalid. Ask them to explain precisely why the conclusion doesn't follow.

CONSOLIDATION

Gallery walk: spotlight the F/H disagreement first. Ask both groups to explain. Then build the definitions table together.

Teacher-Guided Consolidation Notes (≈ 25 minutes)

Build the following table with the class, connecting each definition to the arguments students just sorted. Before each term, ask: "What do you think this word means based on what you just did?"

Term	Definition	Example from today's task
Valid	The conclusion necessarily follows from the premises. IF the premises are true, the conclusion MUST be true. Validity is about structure — not whether the premises actually are true.	<i>F and H: the unicorn and moon-cheese arguments are valid. The logic holds perfectly — the premises just happen to be false.</i>
Sound	Valid AND all premises are actually true. A sound argument guarantees a true conclusion.	<i>A, C, I: the cat/mammal, rain/ground, and Martians arguments — valid structure, true premises.</i>
Invalid	The conclusion does NOT necessarily follow from the premises — the structure is wrong, regardless of whether the premises are true.	<i>B, D, E: the dogs/cats/animals, broccoli/health, and Socrates/mortal arguments — the conclusions don't follow from the premises.</i>

The key point to state directly

Validity is entirely about structure. An argument can be valid even if its premises are false — even if its conclusion is false. What makes an argument valid is that the conclusion must follow from the premises. Students who conflate 'valid' with 'correct' will struggle with proof later.

After building the definitions, display the argument reference table below for students to verify their placements. We will add the last column next class.

ID	Premises	Conclusion	Valid/Sound/Invalid	Structure
A	All cats are mammals. Felix is a cat.	Felix is a mammal.	Sound	Law of Detachment
B	All dogs are animals. All cats are animals.	All cats are dogs.	Invalid	Undistributed Middle
C	If it rains, the ground gets wet. It is raining.	The ground is wet.	Sound	Law of Detachment
D	If I eat broccoli, I'll be healthy. I am healthy.	I must have eaten broccoli.	Invalid	Affirming the Consequent
E	All humans are mortal. Socrates is mortal.	Socrates is a human.	Invalid	Affirming the Consequent
F	All unicorns have horns. Steve is a unicorn.	Steve has a horn.	Valid (not sound)	Law of Detachment

G	If aliens exist, Area 51 is hiding them. Aliens exist.	Area 51 is hiding them.	Valid (conditionally)	Law of Detachment
H	If the moon is made of cheese, it tastes delicious. It is.	It tastes delicious.	Valid (not sound)	Law of Detachment
I	All Martians wear hats. Fred is a Martian.	Fred wears a hat.	Sound (hypothetically)	Law of Detachment

 Students make notes on:

- Definitions: valid, sound, invalid — with one example each from the sorting task
- The key insight: validity is structural, not factual