

Check Your Understanding 7.3

Basic	Intermediate	Advanced
Row 1 Spiral — Trig Ratios → <i>Discovering the invariants (final appearance)</i>		
<i>Basic — Spiral</i> 1. A right triangle has a 67° angle and hypotenuse 1. (a) Express the legs using trig ratios. (b) Apply the Pythagorean theorem. What identity results?	<i>Intermediate — Spiral</i> 2. Two right triangles each have a 33° angle. One has hypotenuse 7, the other 21. (a) Without a calculator, find the ratio of their opposite sides. (b) Show that $\sin 33^\circ$ is the same in both and explain why.	<i>Advanced — Spiral</i> 3. Using a right triangle with hypotenuse 1, prove $\sin^2\theta + \cos^2\theta = 1$ for any acute θ . Verify for $\theta = 30^\circ$ with exact values.
Row 2 Spiral — Solving for Missing Sides → <i>Using invariants</i>		
<i>Basic — Spiral</i> 4. Estimate first, then solve. (a) Right triangle: 46° angle, hyp = 20 m. Find both legs. (b) A flagpole casts a 12 m shadow when the sun is at 41° elevation. Find the height.	<i>Intermediate — Spiral</i> 5. A road descends at 5° for 3 km along the road surface. How far has the driver descended vertically? How far horizontally? 6. Two observers 500 m apart on flat ground sight the top of a tower on opposite sides. Elevation angles are 28° and 42° . Find the tower height.	<i>Advanced — Spiral</i> 7. From a cliff, the angle of depression to a boat is 23° . From a point 30 m higher on the same cliff, the angle of depression to the same boat is 31° . Find the height of the lower observation point above sea level.
Solving for Missing Angles → <i>Inverting the function</i>		
<i>Basic</i> 8. Estimate using the class table first, then find exactly. (a) Right triangle: opp = 5, hyp = 13. Find the angle opposite the side of length 5. (b) Right triangle: adj = 8, hyp = 17. Find the angle adjacent to the side of length 8. (c) A ramp rises 0.8 m over 6 m horizontal. Find the angle of inclination.	<i>Intermediate</i> 9. A ladder 9 m long leans against a wall. The base is 3.5 m from the wall. Find the angle the ladder makes with the ground. 10. A street rises 5 m per 80 m horizontal. Find the angle. Compare to an avalanche-safe slope. 11. $\sin \theta = 0.45$. Estimate θ from the class table, then find exactly. Find $\cos \theta$ and $\tan \theta$ without a calculator.	<i>Advanced</i> 12. Explain $\sin^{-1}$ as a function: its input, output, and why it is only reliable for acute angles in this unit. 13. A student gets $\sin^{-1}(1.05)$ and sees an error. Explain why this is a mathematical impossibility, not a broken calculator.
Algebra Spiral Solve equations	(a) $9x - 5 = 4x + 20$ (b) $3(4y - 2) = 6y + 24$	

Answers — CYU 7.3**Row 1 Spiral (final)**

- 1a.** $\text{opp} = \sin 67^\circ$, $\text{adj} = \cos 67^\circ$.
- 1b.** $(\sin 67^\circ)^2 + (\cos 67^\circ)^2 = 1$. Pythagorean Identity.
- 2a.** 3 (same as ratio of hypotenuses, by similarity).
- 2b.** Both similar by AA. opp/hyp ratio is the same in each triangle individually since they're similar.
- 3.** Hyp = 1, legs = $\sin \theta$, $\cos \theta$. Pythagoras: $\sin^2 \theta + \cos^2 \theta = 1$. Verify 30° : $(1/2)^2 + (\sqrt{3}/2)^2 = 1/4 + 3/4 = 1$. ✓

Row 2 Spiral

- 4a.** $\text{opp} = 20 \sin 46^\circ \approx 14.39$ m, $\text{adj} = 20 \cos 46^\circ \approx 13.89$ m.
- 4b.** $12 \tan 41^\circ \approx 10.44$ m.
- 5.** Vert = $3000 \sin 5^\circ \approx 261$ m. Horiz = $3000 \cos 5^\circ \approx 2989$ m.
- 6.** $h/\tan 28^\circ + h/\tan 42^\circ = 500$. $h(1/\tan 28^\circ + 1/\tan 42^\circ) = 500$. $h \approx 167$ m.
- 7.** Let h = lower height, d = horizontal distance. $\tan 23^\circ = h/d$, $\tan 31^\circ = (h+30)/d$. $h = 30 \cdot \tan 23^\circ / (\tan 31^\circ - \tan 23^\circ) \approx 72$ m.

Solving for Missing Angles

- 8a.** $\sin^{-1}(5/13) \approx 22.6^\circ$.
- 8b.** $\cos^{-1}(8/17) \approx 61.9^\circ$.
- 8c.** $\tan^{-1}(0.8/6) \approx 7.6^\circ$.
- 9.** $\cos^{-1}(3.5/9) \approx 67.1^\circ$.
- 10.** $\tan^{-1}(5/80) \approx 3.6^\circ$. Well below the 30° avalanche danger lower bound.
- 11.** $\theta = \sin^{-1}(0.45) \approx 26.7^\circ$. $\cos \theta \approx 0.893$. $\tan \theta \approx 0.504$.
- 12.** $\sin^{-1}$ takes a ratio 0–1 and outputs the acute angle. Reliable only for 0° – 90° since the definition uses right triangles.
- 13.** $\text{opp/hyp} < 1$ always. No acute angle has sine 1.05 — it violates right triangle geometry.

Algebra Spiral

- (a)** $x = 5$
- (b)** $y = 5$