Unit 4

Triangle Theorems

6 lessons • CYU replaces homework throughout

Unit Navigation Instrument

Big Idea: Triangles obey laws. Those laws can be proven — and once proven, they become tools for solving problems and building further proofs.

Subtopic	Basic	Intermediate	Advanced
Triangle and Polygon Angle Relationships *→ Proving the laws*	State the triangle angle sum theorem and the exterior angle theorem as conditional statements. State the polygon interior angle sum formula.	Use the triangle angle sum theorem, exterior angle theorem, or polygon formula to find missing angles and solve for variables.	Prove the triangle angle sum theorem. Derive the polygon interior angle sum formula and explain why two different methods give the same result.
Constructions *→ Building the tools*	Describe the steps to construct an angle bisector, perpendicular bisector, or altitude using only compass and straightedge.	Perform a construction correctly. Label it and identify the triangle congruence that guarantees it works.	Justify why a construction produces the correct result. Write the proof that the construction does what it claims.
Base Angles Theorem *→ Proving and earning the converse*	State the base angles theorem and its converse. Identify which applies in a given situation.	Use the base angles theorem or its converse to find missing sides or angles and solve algebraic problems.	Prove the base angles theorem from triangle congruence. Explain why the converse requires a separate proof — and why together they form a biconditional.
Bisector Theorems *→ Turning constructions into proofs*	State the angle bisector theorem and the perpendicular bisector theorem as conditional statements. State their converses.	Use either theorem or its converse to find missing measurements and set up algebraic equations.	Prove a result by invoking a bisector theorem as a step in a longer argument. Use the converse to justify a claim about a point's location.

The Advanced column throughout this unit is proof or proof-adjacent. This is intentional. Knowing why a theorem is true is what makes it a reliable tool, not just a memorized fact. The base angles row includes the biconditional explicitly: students who reach Advanced understand that the theorem and its converse are separate claims requiring separate proofs, and that once both are established they can be applied in either direction.

Unit 3 established the architecture of proof: flow chart and paragraph proofs, the law of syllogism as the visible shape of a deductive chain, CPCTC as a policy not just a label. This unit uses that architecture to prove new geometric results — results about triangles that students may already believe but have not yet established from axioms.

The unit has two layers. The first is content: the triangle angle sum theorem, the exterior angle theorem, polygon angle sums, constructions for copying angles and building parallel lines and altitudes, the base angles theorem and its converse, and the angle bisector and perpendicular bisector theorems. These are the facts students will draw from in proof writing for the rest of the course.

The second layer is epistemological, and it is the one that makes this unit worth teaching the way it is taught. In Lesson 4.1 students will conjecture that triangle angles sum to 180° and attempt to prove it — and they will find that every proof path they can imagine requires a construction that has not yet been established. In Lesson 4.2 they build those constructions, close the logical gap, and complete the chain. That experience — of carrying an open conditional for a full lesson before seeing it resolved — is not a teaching inefficiency. It is the lesson. It is what mathematicians actually do.

The unit was largely untaught in this format Year 1 due to the timing of holidays, midterms, and the end of the first semester. What was taught was taught directly rather than through the task structure described here. The lesson designs are therefore newer and less tested than those in Units 1–3. Teachers should expect to iterate on them and are encouraged to bring their own notes back to future versions of this chapter.

Lesson Sequence Overview

Lesson 4.1: Triangle Angle Sum, Exterior Angles, and Polygon Sums — conjecture, attempted proof, honest accounting of dependencies, three payoffs

Lesson 4.2: Three Constructions and Their Proofs — angle copy, parallel through a point, altitude — closure of the 4.1 chain

Lesson 4.3: The Base Angles Theorem and Its Converse — two hikers, angle bisector proof, biconditional

Lesson 4.4: The Bisector Theorems — search-and-rescue beacon, water tower, angle bisector and perpendicular bisector theorems

Lesson 4.5: Practice Day — individual work anchored to navigation instrument, teacher circulates and confers

Lesson 4.6: Assessment