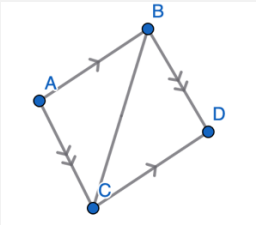
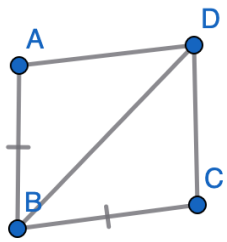
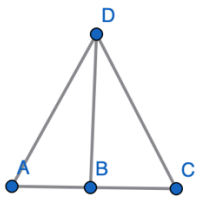
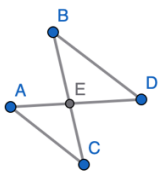
Check Your Understanding 3.5a

Complete each problem. Write CPCTC in full before using it. Flow chart or paragraph proof only — no two-column proofs.

Basic	Intermediate	Advanced
Row 3 Spiral — Triangle Congruence Shortcuts *→ The geometric facts we draw from to build arguments*		
Basic — Spiral State the additional information needed to prove each pair of triangles congruent by the given shortcut. *(a) ASA: △ABC and △DEF, with ∠A = ∠D and AB = DE. What else is needed?* *(b) SAS: △PQR and △XYZ, with PQ = XY and PR = XZ. What else is needed?*	*Intermediate — Spiral*  List every pair of congruent parts you can establish from the given and the diagram. For each, state whether it is given, free from the diagram, or derived from a theorem. Which shortcut proves △ABC ≅ △CDA?	*Advanced — Spiral* A student proves △ABD ≅ △CBD by ASA. The given information is: AB = CB, ∠ADB = ∠CDB, and BD is shared. *(a) Is the student's shortcut correct? If not, what shortcut does this information actually support?* *(b) Write the complete congruence statement with correct vertex correspondence.* *(c) State one pair of parts that follow from CPCTC. Write the full definition before using it.*
Row 4 Spiral — Reading a Diagram *→ Knowing what the diagram gives us for free*		
Basic — Spiral Examine the diagram. Mark all free information directly on it. Then list what is given and what is free. 	*Intermediate — Spiral* A student is working with the diagram below. They mark ∠ABD = ∠CBD because 'the diagram looks symmetric.' Is this valid free information? Explain why or why not, and state what they would need instead. 	*Advanced — Spiral* Examine the diagram. List all free information. Then explain: what is the minimum additional given information required to prove the two triangles congruent? Give the most efficient possible set of givens and state the shortcut. 

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Writing Proofs

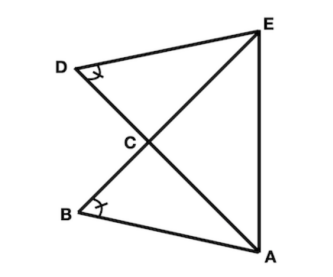
→ *Constructing arguments that cannot reasonably be rejected*

Basic

Basic — Scaffolded Proof

Given: ∠D ≅ ∠B, ∠DEA ≅ ∠BAE.

Prove: △BEA ≅ △DAE.



The proof steps are given below. Fill in the missing reason for each step.

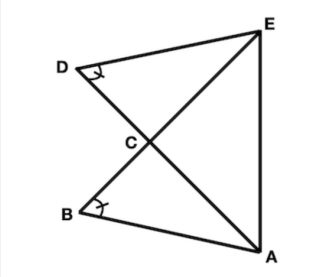
Statement	Reason
∠D ≅ ∠B	_____
∠DEA ≅ ∠BAE	_____
AE ≅ AE	_____
△BEA ≅ △DAE	_____

Intermediate

Intermediate — Complete Proof

Given: ∠D ≅ ∠B, ∠DEA ≅ ∠BAE.

Prove: △BCA ≅ △DCE.



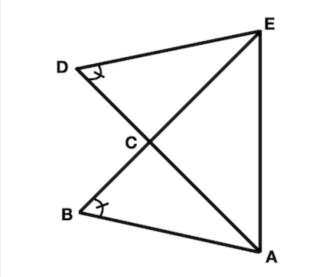
Hint: you will need the Basic result as an intermediate step.

Advanced

Advanced — Multi-Step Proof

Given: ∠D ≅ ∠B, ∠DEA ≅ ∠BAE.

Prove: DC ≅ BC.



Plan your proof before writing it. What intermediate results do you need?

Planning Proofs

→ *Seeing the path before writing the argument*

Basic

Given: ABCD is a parallelogram. Diagonal AC is drawn.

(a) Without writing the proof, identify the key intermediate step you would need to establish first.

(b) What shortcut would you use? What three pairs of congruent parts would you need?

Intermediate

Plan the following proof using backward reasoning. Do not write the proof yet.

Given: ABCD is a parallelogram. Diagonal AC is drawn.

To prove: AB = CD and BC = AD.

Start from the conclusion and work backward: what would you need to know one step before? Two steps before? Keep going until you reach the given.

Advanced

Given: ABCD is a parallelogram. Diagonal AC is drawn.

Prove: AB = CD and BC = AD.

Write the complete proof. Use the plan you developed at Intermediate level.

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Basic	Intermediate	Advanced
Algebra Spiral *Factor*	*Algebra Spiral* Factor each expression completely. *(a) $x^2 + 7x + 12$* *(b) $2a^2 - 8a$* *(c) $3y^2 - 75$*	

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CYU 3.5a — Answer Key (Teacher Copy)

Row 3 Spiral

(a) ∠B = ∠E.

(b) ∠P = ∠X (the included angle between the two given sides).

Intermediate: AB ∥ CD → ∠BAC = ∠DCA (alt. int. angles, derived). BC ∥ AD → ∠ABC = ∠CDA (alt. int. angles, derived). AC = CA (reflexive, free). Three pairs established. Shortcut: ASA — △ABC ≅ △CDA using ∠BAC = ∠DCA, AC = CA, ∠BCA = ∠DAC.

Adv (a): Not ASA — the given includes a side (AB = CB) not two angles and an included side. The correct shortcut is SAS if ∠ABD = ∠CBD is given, or SSS if a third side is established.

Adv (b): With AB = CB, ∠ADB = ∠CDB, BD = BD: △ABD ≅ △CBD by SAS.

Adv (c): AD = CD (CPCTC = Corresponding Parts of Congruent Triangles are Congruent).

Row 4 Spiral

Basic: Free: BD = BD (reflexive). Given: AB = CB (tick marks). Nothing else is free without parallel lines or angle marks.

Intermediate: Not valid free information. The diagram looking symmetric is not a valid geometric argument. They would need an angle bisector given, or ∠ABD = ∠CBD marked.

Advanced: Free: vertical angles at E (∠AEC = ∠BED). Minimum additional given will vary, but two things will be needed..

Writing Proofs (Row 5)

Basic reasons: Given; Given; Reflexive property; AAS.

Intermediate: First establish △BEA ≅ △DAE (Basic proof). By CPCTC (Corresponding Parts of Congruent Triangles are Congruent): BE = DA and EA = AE. Then ∠ABE = ∠CDA and ∠BAE = ∠DAE. Use angle relationships to establish parts needed for △BCA ≅ △DCE by AAS.

Advanced: Chain: Basic → △BEA ≅ △DAE → CPCTC → Intermediate → △BCA ≅ △DCE → CPCTC → DC = BC.

Planning Proofs (Row 6)

Basic: Key intermediate step: prove △ABC ≅ △CDA. Need: two pairs of alternate interior angles (from AB ∥ CD and BC ∥ AD) plus reflexive side AC. ASA.

Intermediate: Working backward: want AB = CD and BC = AD → need CPCTC → need △ABC ≅ △CDA → need two angle pairs and side AC → alternate interior angles from the parallel sides → this follows from the definition of parallelogram. Chain complete.

Advanced: AB ∥ DC → ∠BAC = ∠DCA (alt. int. angles). BC ∥ AD → ∠BCA = ∠DAC (alt. int. angles). AC = CA (reflexive). △ABC ≅ △CDA (ASA). AB = CD and BC = AD (CPCTC). ■

Algebra Spiral

(a) (x + 3)(x + 4)

(b) 2a(a − 4)

(c) 3(y − 5)(y + 5)

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