Unit 1 — Differentiation Day Practice

# Lessons 1.1 – 1.3

Name: ______________________________

Period: ________ Date__________

*Show all work. Use correct notation throughout.*

| What Makes a Good Definition<br>*→ Classification and differentiation* | Basic | Intermediate | Advanced |
|---|---|---|---|

**1** *Basic*

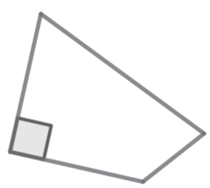
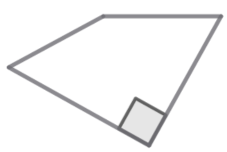
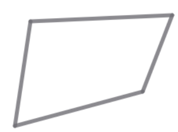
A student defines a triangle as: ‘A polygon with three angles.’

(a) What does this definition classify correctly?

(b) Does it differentiate? Could any non-triangle satisfy this definition? Explain.

**2** *Intermediate*

Here are four figures. Two are ‘widgets’ and two are not. Study the examples and non-examples. Write a definition for ‘widget’ using classification and differentiation. Then test your definition — does it include both widgets and exclude both non-widgets?

| Widgets | Non-widgets |
|---|---|
| <br> | <br> |

**3** *Advanced*

A student defines a ‘teenager’ as: ‘A person who is between 13 and 19 years old.’

Another student argues that a 13-year-old who turns 14 tomorrow is ‘barely a teenager’ and shouldn’t count.

Does the definition need revision, or is the second student confused about how definitions work? Explain using what you know about classification and differentiation.

| **The Foundational Objects** *→ Naming what we reason about* | **Basic** | **Intermediate** | **Advanced** |
|---|---|---|---|

**4** *Basic*

Draw a single diagram that contains all of the following. Label each one with its name and correct notation. Use exactly five labeled points.

- A line
- A ray (that is not part of the line above)
- A segment
- A plane

**5** *Intermediate*

Points A, B, and C are collinear. Points A, B, and D are not collinear.

(a) Can points A, B, C, and D all be coplanar? Must they be? Explain.

(b) Draw a diagram that supports your answer.

**6** *Advanced*

A student says: 'I can define a line as the shortest path between two points.'

(a) Evaluate this claim. Does it classify and differentiate correctly?

(b) Does this definition work on the surface of a globe? What does that tell you?

(c) Connect this to why 'line' is left as an undefined term in Euclidean geometry.

| **Distance and Midpoint in the Plane** *→ Measuring what we reason about* | **Basic** | **Intermediate** | **Advanced** |
|---|---|---|---|

**7** *Basic*

(a) Find the distance between $(-1, 3)$ and $(5, -5)$.

(b) Find the midpoint of the segment from $(-1, 3)$ to $(5, -5)$.

**8** *Intermediate*

(a) The midpoint of segment CD is $M(4, -2)$. Point C is at $(7, 1)$. Find D.

(b) Are the points $(2, 1)$, $(6, 3)$, and $(10, 5)$ collinear? Justify your answer using slopes or distances.

**9** *Advanced*

Two hikers start at point $A(0, 0)$. One walks to $B(12, 0)$ and the other walks to $C(0, 16)$. A helicopter can land exactly at the midpoint of segment BC.

(a) Find the coordinates of the helicopter's landing point.

(b) How far is the helicopter from each hiker (A, B, and C)?

(c) Is the helicopter equidistant from B and C? Must it always be when landing at the midpoint of a segment? Explain why.

**10** *Basic*

(a) A quadrilateral has vertices A(1, 1), B(7, 1), C(7, 4), D(1, 4). Find its area and classify it by number of sides and concavity.

(b) A triangle has vertices P(0, 0), Q(10, 0), R(4, 3). Find its area.

**11** *Intermediate*

A pentagon has vertices at A(0, 0), B(6, 0), C(6, 3), D(3, 5), E(0, 3).

(a) Find the perimeter. Show your distance formula work for each side.

(b) For which sides did you need the distance formula, and which could you find by counting? Explain why.

**12** *Advanced*

A student says: ‘The perimeter of a triangle with vertices (0, 0), (3, 0), and (0, 4) is 3 + 4 + 5 = 12. I found the slanted side by counting diagonals on the grid.’

The answer is correct, but the reasoning is flawed.

(a) Explain what is wrong with ‘counting diagonals’ as a general method.

(b) Describe a specific triangle where counting diagonals would give the wrong perimeter.

## Answers — Differentiation Day Practice

### Row 1 — What Makes a Good Definition

1. (a) It classifies correctly — a triangle is a polygon. (b) It differentiates partially: ‘three angles’ is correct, but every polygon has as many angles as sides, so ‘a polygon with three sides’ or ‘three angles’ both work. The definition is acceptable but could be made more standard by saying ‘three sides.’ The key test: does any non-triangle satisfy it? No — the only polygon with three angles is a triangle.

2. A widget is a convex quadrilateral (classification) with exactly one right angle (differentiation). Test: both widgets have four sides, are convex, and have exactly one 90° angle. The non-widget quadrilateral has no right angle — excluded. The right triangle has only three sides — excluded. Definition works.

3. The definition does not need revision. ‘Between 13 and 19’ is precise — the 13-year-old satisfies the definition regardless of when their birthday is. The second student is confusing the boundary of a definition with a gradation. Definitions draw sharp lines; that is what makes them useful. A good definition includes all examples and excludes all non-examples. It does not have grey zones.

### Row 2 — The Foundational Objects

4. Accept any diagram with all four objects correctly drawn and labeled with notation. Line: double arrow, e.g. line AB. Ray: single arrow from endpoint, e.g. ray CD. Segment: bar notation, e.g. segment EF (or reuse points). Plane labeled with three non-collinear points.

5. (a) Yes, they can all be coplanar — A, B, C lie on a line, and D is off that line but can still be in the same plane. In fact they must be coplanar: any three non-collinear points (e.g. A, B, D) determine exactly one plane, and C, being on line AB, lies in that plane automatically. (b) Diagram: a line with A, B, C on it and D off the line, all drawn on a flat surface.

6. (a) It classifies ('path') and differentiates ('shortest'). On a flat plane, this correctly identifies a straight line. (b) On a globe, the 'shortest path' between two points is a great circle arc, which is curved — not what we call a 'line' in Euclidean geometry. (c) This shows that any attempt to define 'line' either works only in certain contexts or requires other terms ('straight,' 'flat') that are equally basic. This circularity is why 'line' is left undefined.

### Row 3 — Distance and Midpoint

7. (a) $d = \sqrt{[(5-(-1))^2 + (-5-3)^2]} = \sqrt{[36+64]} = \sqrt{100} = 10$. (b) $M = ((-1+5)/2, (3+(-5))/2) = (2, -1)$.

8. (a) $M = ((7+x)/2, (1+y)/2) = (4, -2)$. So $x = 1$, $y = -5$. $D = (1, -5)$. (b) Slope $(2,1)\rightarrow(6,3) = 2/4 = 1/2$. Slope $(6,3)\rightarrow(10,5) = 2/4 = 1/2$. Equal slopes through a shared point → collinear.

9. (a) Midpoint of BC = $((12+0)/2, (0+16)/2) = (6, 8)$. (b) Distance to A: $\sqrt{(36+64)} = 10$. Distance to B: $\sqrt{(36+64)} = 10$. Distance to C: $\sqrt{(36+64)} = 10$. The helicopter is equidistant from all three hikers — 10 units from each. (c) Yes, the midpoint of a segment is always equidistant from both endpoints — by definition, it splits the segment into two equal halves.

### Row 4 — Polygons and Area

10. (a) Convex quadrilateral. Area = $6 \times 3 = 18$ square units. (b) Base = 10, height = 3. Area = $\frac{1}{2} \times 10 \times 3 = 15$ square units.

11. (a) AB = 6 (horizontal). BC = 3 (vertical). $CD = \sqrt{[(6-3)^2+(3-5)^2]} = \sqrt{[9+4]} = \sqrt{13}$. $DE = \sqrt{[(3-0)^2+(5-3)^2]} = \sqrt{[9+4]} = \sqrt{13}$. EA = 3 (vertical). Perimeter = $6 + 3 + \sqrt{13} + \sqrt{13} + 3 = 12 + 2\sqrt{13} \approx 19.21$. (b) AB and BC are horizontal/vertical — their lengths can be found by counting grid units. CD and DE are slanted — they require the distance formula because counting grid squares only measures horizontal or vertical distance, not the actual diagonal length.

12. (a) 'Counting diagonals' works only when the slanted side happens to form a Pythagorean triple with integer legs. The student got lucky: legs 3 and 4 give hypotenuse 5. But the method has no mathematical basis — it does not generalize. (b) Triangle with vertices (0,0), (2,0), (0,3). The slant side has length $\sqrt{(4+9)} = \sqrt{13} \approx 3.606$. Counting diagonals on the grid would not yield this value.

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