Lesson 4.2 — Teacher Reference Notes

Three Constructions and Their Proofs

Unit 4: Triangle Theorems • Day 2 of 6

At a Glance	
	Pacing: Tasks 1–3 board work (~50 min, given individually as groups are ready) • Gallery walk if time allows • Teacher-guided notes (~30 min, priority) • CYU begins if time allows **Materials:** Compass and straightedge for every student • Large demonstration compass • Paper on tables (Tasks 1–3 done on paper, not boards) • Task 3 printed card — altitude construction (print a few copies before class) • Student note sheets (distribute for notes phase) **Nav instrument rows active:** Row 2 (Constructions) activates today • Row 1 Advanced closes — the full deductive chain from Lesson 4.1 is now complete **Priority if time runs short:** Notes first. The closure moment (naming the complete chain) must happen. Gallery walk is optional. CYU is homework. **Task 3 note:** Print the altitude construction card before class. The steps are too detailed to dictate while managing groups. The full proof appears in teacher-guided notes regardless of how far groups get.

OBJECTIVES

By the end of this lesson students can:

- Construct a copy of an angle using compass and straightedge and prove it works using SSS and CPCTC.
- Construct a line through a given point parallel to a given line and prove it works using the converse of the alternate interior angles theorem.
- Explain that Propositions 23 and 31 together complete the deductive chain begun in Lesson 4.1.
- (Extension) Construct an altitude of a triangle and prove it meets the base at 90° using SSS→CPCTC→SAS→CPCTC→linear pair.

The anticlimactic quality — name it honestly

When students find the angle copy construction, the response is often disappointment. ‘It’s just copying lengths?’ Name this honestly: the interesting mathematical work was establishing Propositions 2 and 3 in the first place. The construction is their payoff. Students who feel let down by the simplicity of the method often find the proof more satisfying — the SSS argument is where the real thinking lives.

BOARD TASKS (≈50 MIN, GIVEN INDIVIDUALLY AS GROUPS ARE READY)

All three tasks are done on paper with compass and straightedge — not at the boards with markers. Paper is significantly better for precision. Boards can be used to write the proofs.

Task 1 — Copy an Angle (Proposition 23)

Prompt

Draw an angle of your choice. Draw a separate ray somewhere else on your paper. Using only a compass and straightedge — and only moves that can be justified by our postulates and propositions — construct an exact copy of your angle on the separate ray. Once you have a construction that works, write a proof that it produces an equal angle. Every step of your construction needs a justification.

⚠️ What to watch for

Expected approach: Place compass at vertex, draw arc crossing both rays (label B, C). Transfer same radius to new ray (label B'). Measure chord BC, transfer to B' (label C'). Draw ray through C'. Proof: SSS on the two triangles formed (two equal radii + chord length) → CPCTC gives equal angles.

Common error: Students measure the angle with a protractor and redraw it. Redirect: 'Which postulate allows a protractor measurement? Can you find one in our list?'

Anticlimactic moment: Groups who say 'that's it?' — see the note above. The proof is where the interest lives, not the construction steps.

Task 2 — Parallel Line Through a Given Point (Proposition 31)

Prompt

Draw a line and a point not on that line on your paper. Using only compass and straightedge, construct a line through that point parallel to the given line. Use only moves justified by our postulates and propositions. Once you have a construction, prove that the line you built is actually parallel to the given line. What theorem from Unit 3 makes your proof work?

The standard method (Proposition 31)

- Draw a transversal through point P, crossing line l at point Q.
- At P, copy the angle that the transversal makes with l — placing it on the opposite side of the transversal. This creates alternate interior angles.
- The ray through P at the copied angle is the parallel line.

Why opposite sides: the angle must be placed as an alternate interior angle — on the opposite side of the transversal, between the two lines. Copying on the same side gives co-interior angles, which only guarantee parallel lines when they sum to 180° — not true for an arbitrary angle.

Proof: the copied angle equals the original by Task 1 (CPCTC). Equal alternate interior angles → parallel lines by the converse of the alternate interior angles theorem, Unit 3.

Task 3 — Altitude of a Triangle (Proposition 12) — Extension

Print the task card before class and distribute individually to groups that have completed Tasks 1 and 2. Intended for all groups — those that don't reach it will see the proof in notes.

Construction steps (from printed card)

- Place compass needle at vertex A. Open wide enough to cross BC at two points. Label P and Q.
- From P, make an arc on the far side of BC.
- From Q, same compass width, make an arc crossing the first. Label the intersection R.
- Draw line AR. Label where it meets BC as H. AH is the altitude.

Now, prove you have created a line from a that is perpendicular to BC.

⚠ The proof chains every major Unit 3 tool in sequence

SSS → CPCTC → SAS → CPCTC → linear pair. Students who worked through it will have experienced the satisfaction of a multi-step proof coming together. The complete proof is in the teacher reference below.

Prompt if groups get stuck after the first SSS step: 'You've proven the large triangles are congruent. What did CPCTC give you? Can you use that result to compare a new pair of smaller triangles?'

THE CLOSURE MOMENT — SAY THIS EXPLICITLY DURING NOTES

Narrative Arc — After establishing the parallel line construction

"In Lesson 4.1 we wrote a conditional proof: if we can construct a line through any point parallel to any given line, the triangle angle sum must be 180°. But that construction required copying an angle — which we also had to prove we could do. Today we established both. Copying an angle is Proposition 23. The parallel line construction is Proposition 31. Together they close the chain: Proposition 23 feeds into 31, which feeds into 32 — the triangle angle sum. And from 32, the exterior angle theorem and the polygon angle sum formula follow immediately. Everything from Lesson 4.1 is now fully proven from our axioms."

"This moment is worth a full pause. Students have been carrying an open conditional since Lesson 4.1. Name it explicitly as a genuine event: the chain is closed."

CONSTRUCTION REFERENCE — ALL THREE (TEACHER COPY)

Construction 1 — Copy an Angle (Proposition 23)

#	Construction step	Justification (postulate or proposition)
1	Place compass at vertex of original angle. Draw arc crossing both sides. Label intersections B and C.	*Postulate 3: a circle with any center and any radius.*
2	On new ray, place compass at starting point. Draw arc of the same radius. Label intersection B'.	*Postulate 3; Proposition 3: copy the same radius.*
3	Set compass to the chord length BC.	*Proposition 2/3: measure and transfer a length.*
4	Needle at B'. Draw arc of chord length. Label intersection C'.	*Postulate 3.*
5	Draw ray from starting point through C'.	*Postulate 1: a line through any two points.*

Proof that the construction works:

Statement	Justification
AP = AB and A'P' = A'B'	*Equal compass radii (Propositions 2 and 3).*
PC = BC and P'C' = B'C'	*Chord length transferred — equal compass radii.*
AC = A'C'	*Same arc radius used in Steps 1 and 2.*
△APC ≅ △A'P'C'	*SSS.*
∠A ≅ ∠A' ■	*CPCTC.*

Proposition 23 (conditional form): If we have an angle and a ray, we can construct a copy of the angle on that ray.

Construction 2 — Parallel Line Through a Point (Proposition 31)

#	Construction step	Justification (postulate or proposition)
1	Draw a transversal through P, crossing line l at Q.	*Postulate 1.*
2	At P, copy the angle the transversal makes with l — on the opposite side of the transversal.	*Construction 1 (Proposition 23), just established.*
3	The ray through P is the parallel line.	*See proof below.*

Proof that the construction works:

Statement	Justification
∠QPl ≅ ∠QP'l'	*Angle copy construction — CPCTC (Proposition 23).*
These are alternate interior angles of	*They are on opposite sides of PQ, between the two lines.*

transversal PQ cutting l and l'.	
l ‖ l' ■	*Converse of the alternate interior angles theorem (Unit 3).*

Proposition 31 (conditional form): If we have a line and a point not on it, we can construct a line through the point parallel to the given line.

Construction 3 — Altitude of a Triangle (Proposition 12)

#	Construction step	Justification (postulate or proposition)
1	Needle at vertex A. Open wide enough to cross BC at two points. Label them P and Q.	*Postulate 3.*
2	From P, make an arc on the far side of BC.	*Postulate 3.*
3	From Q, same width, make an arc crossing the first. Label intersection R.	*Postulate 3; Proposition 3.*
4	Draw line AR. Label where it meets BC as H. (H is the foot of the altitude.)	*Postulate 1.*

Proof that the construction works:

Statement	Justification
AP = AQ	*Equal compass radii — Steps 1 (Propositions 2/3).*
PR = QR	*Equal compass radii — same width from P and Q.*
AR = AR	*Reflexive property.*
△APR ≅ △AQR	*SSS.*
∠PAR ≅ ∠QAR	*CPCTC — the angle at A is bisected by AR.*
AP = AQ, ∠PAH ≅ ∠QAH, AH = AH	*Step above; ∠PAH = ∠QAH (same as ∠PAR = ∠QAR); AH reflexive.*
△APH ≅ △AQH	*SAS.*
∠AHP ≅ ∠AHQ	*CPCTC.*
∠AHP + ∠AHQ = 180°	*Linear pair — H lies on BC.*
∠AHP = ∠AHQ = 90°	*Two congruent angles summing to 180° must each be 90°.*
AH ⊥ BC ■	*Definition of perpendicular.*

Proposition 12: An altitude from a vertex meets the opposite side (or its extension) at a right angle. The proof chains SSS → CPCTC → SAS → CPCTC → linear pair — every major Unit 3 tool in one argument.

STUDENTS MAKE NOTES ON

✎ **Students make notes on:**

- Construction 1: Copy an Angle — steps with justifications, SSS proof, conditional form (Proposition 23)
- Construction 2: Parallel Line Through a Point — steps with justifications, alternate interior angles proof, conditional form (Proposition 31)
- The closure moment: Prop 23 → Prop 31 → Prop 32 (triangle angle sum). The full chain is now closed from the axioms.
- Construction 3 (if time allows): Altitude — steps, SSS→CPCTC→SAS→CPCTC→linear pair proof, definition of altitude

Connection Points	• Draws from Lesson 4.1: the conditional proof from Lesson 4.1 is completed here — name this explicitly • Draws from Unit 3: SSS, SAS, CPCTC, and the alternate interior angles theorem (and its converse) are all used in the proofs today • Draws from Lesson 3.1: the same postulates (1, 2, 3) that justified the equilateral triangle construction in Unit 3 appear here again • Feeds into Lesson 4.3: the altitude definition introduced here is the foundation for perpendicular distance reasoning in Lesson 4.4