Check Your Understanding 8.6

Show all work and reasoning. Rows 1 and 2 have retired from the spiral; the remaining rows continue.

Basic	Intermediate	Advanced
Row 3 Spiral — Arc length and sector area → *Applying proportional reasoning*		
Basic — Spiral **1.** A triangle is inscribed in a circle of radius 5; one angle is 72°. What central angle subtends the arc opposite that angle?	*Intermediate — Spiral* **2.** Find the length of that arc.	*Advanced — Spiral* **3.** Find the sector area for that arc, then the area of the triangle formed by the two radii and the chord. What fraction of the sector is the triangle?
Row 4 Spiral — Central and inscribed angles → *The spine of the unit*		
Basic — Spiral **4.** A triangle inscribed in a circle has angles 50°, 60°, 70° and circumradius $R = 8$. Find the three arc lengths (in terms of π).	*Intermediate — Spiral* **5.** Verify that the three arcs sum to the full circumference.	*Advanced — Spiral* **6.** Find all three side lengths using the chord-sine identity, $a = 2R \sin A$.
Row 5 Spiral — Tangent lines and chord theorems → *Consequences of the definition*		
Basic — Spiral **7.** Triangle ABC is inscribed in a circle; the tangent at A makes a 55° angle with chord AB. Find arc AB, and explain.	*Intermediate — Spiral* **8.** Using the tangent-chord theorem (tangent-chord angle = inscribed angle in the alternate segment), find angle ACB.	*Advanced — Spiral* **9.** If the circumradius is 10, find side AB.
Circles and triangles: closing the loop → *Synthesis with Units 6 and 7*		
Basic **10.** Triangle ABC has side $a = 10$ and angle $A = 30°$. (a) Find the circumradius R. (b) If angle $B = 45°$, find side b. (c) Find the area using Area $= \frac{1}{2}ab \cdot \sin C$, with angle $C = 105°$.	*Intermediate* **11.** A triangle is inscribed in a circle of radius 6. (a) One side has length 6 — what inscribed angle subtends it? (b) Another side subtends a 90° inscribed angle — find its length and say what that tells you about the triangle. **12.** Triangle ABC has $b = 5$, $c = 7$, and angle $A = 40°$. (a) Find the area. (b) Find the circumradius R (use the Law of Sines and the area formula together).	*Advanced* **13.** An equilateral triangle with side length s is inscribed in a circle. (a) Find R in terms of s using the chord-sine identity. (b) Find the area in terms of s via Area $= \frac{1}{2}ab \cdot \sin C$, and verify with base × height / 2. (c) Show $R = s\sqrt{3}/3$, and explain why the circumcenter lies at the centroid, using symmetry.

Algebra Spiral — Exponent Rules

Simplify. Write every answer with positive exponents.

(a) $(x^5y^{-3})^2 \div (x^2y)$

(b) $(4a^3)^2 \cdot (2a)^{-1}$

(c) $(3x^2y^0)^3$

(d) $(m^{-2}n^4)^{-1} \cdot (m^3n)$

Answers — CYU 8.6

Row 3 Spiral — Arc length and sector area

1. $2 \times 72° = 144°$.

2. $(144/360) \cdot 2\pi(5) = 4\pi$.

3. Sector = $(144/360)\pi(25) = 10\pi$. Triangle = $\frac{1}{2} \cdot 5 \cdot 5 \cdot \sin 144° \approx 7.35$. Fraction $\approx 7.35 / (10\pi) \approx 0.23$.

Row 4 Spiral — Central and inscribed angles

4. Central angles 100°, 120°, 140°. Arcs: $40\pi/9$, $16\pi/3$, $56\pi/9$.

5. $40\pi/9 + 48\pi/9 + 56\pi/9 = 144\pi/9 = 16\pi = 2\pi(8)$. ✓

6. $16 \sin 50° \approx 12.26$; $16 \sin 60° = 8\sqrt{3} \approx 13.86$; $16 \sin 70° \approx 15.04$.

Row 5 Spiral — Tangent lines and chord theorems

7. The tangent-chord angle equals half the intercepted arc, so arc AB = $2 \times 55° = 110°$.

8. Angle ACB = 55°.

9. $AB = 2R \sin(ACB) = 20 \sin 55° \approx 16.38$.

Circles and triangles: closing the loop

10. (a) $a = 2R \sin A \rightarrow 10 = 2R(\frac{1}{2}) \rightarrow R = 10$. (b) $b = 2R \sin B = 20 \sin 45° = 10\sqrt{2} \approx 14.14$. (c) Area = $\frac{1}{2} \cdot 10 \cdot 10\sqrt{2} \cdot \sin 105° = 25(\sqrt{3} + 1) \approx 68.3$.

11. (a) $6 = 2(6)\sin\theta \rightarrow \sin\theta = \frac{1}{2} \rightarrow \theta = 30°$. (b) Side = $2(6)\sin 90° = 12$ = the diameter; the triangle has a right angle at the opposite vertex (Thales in reverse).

12. (a) $\frac{1}{2} \cdot 5 \cdot 7 \cdot \sin 40° \approx 11.25$. (b) Law of Cosines gives $a \approx 4.51$; $R = a/(2 \sin A) \approx 3.51$.

13. (a) $s = 2R \sin 60° = R\sqrt{3} \rightarrow R = s/\sqrt{3} = s\sqrt{3}/3$. (b) Area = $\frac{1}{2}s^2 \sin 60° = s^2\sqrt{3}/4$; with $h = s\sqrt{3}/2$, $\frac{1}{2} \cdot s \cdot (s\sqrt{3}/2) = s^2\sqrt{3}/4$. ✓ (c) $R = s\sqrt{3}/3$. By the triangle's three-fold symmetry the circumcenter, centroid, and incenter all lie on every axis of symmetry, so they coincide.

Algebra Spiral — Exponent Rules

(a) x^8/y^7

(b) $8a^5$

(c) $27x^6$

(d) m^5/n^3