

Special Parallelograms: Rectangles, Rhombi, and Squares

Unit 5: Quadrilaterals • Day 3 of unit

At a Glance

Pacing: Prompt 1: Rectangle (~20 min) • Prompt 2: Rhombus (~20 min) • Extension: Square (~10 min) • Gallery walk if time allows (~8 min) • Consolidation: both proofs + square inheritance (~25 min) • Share-and-revise notes (~12 min)

Nav instrument rows active: Row 3 (Special Parallelograms) — all three levels • Row 1 Advanced (property inheritance) continues

Note sheet delivery: Distribute before the task begins. Students record a property only after the teacher has checked their board proof. This keeps notes authentic.

Unit 4 connection: The rhombus proof uses the base angles theorem (Unit 4, Lesson 4.3). Name this explicitly during consolidation.

OBJECTIVES

By the end of this lesson students can:

- State the additional constraint that defines each special parallelogram.
- State which diagonal properties each special parallelogram has.
- Prove the rectangle diagonal theorem (diagonals equal) using SAS and CPCTC.
- Prove the rhombus diagonal theorem (diagonals perpendicular and bisect vertex angles) using the base angles theorem and SSS.
- Explain why the square's properties follow from the rectangle and rhombus results without new proof.

 **Narrative Arc** — Before the first prompt

“In Lesson 5.2 we proved four properties of parallelograms. Those properties belong to every parallelogram — and therefore to every shape in the parallelogram branch of the hierarchy. Today we add one more constraint and ask what new properties emerge. One new condition. What does it force?”

BOARD TASKS

Prompt 1 — Rectangle (~20 min) — Read aloud to all groups simultaneously

Prompt

Your parallelogram has one new condition: all four angles are right angles. Mark what you know. Draw both diagonals. What do you notice? Can you prove it? Write each new result as a conditional statement.

⚠ What to watch for

The key observation: the diagonals appear equal in length. Push for a proof, not just a measurement.

The proof path: SAS on the two triangles formed by one diagonal. The two legs of the right angle are shared sides ($AB = AB$, $BC = BC$) and the included angle is 90° on both sides. Some groups will initially try ASA or SSS — redirect to SAS with the right angle as the included angle.

Redirect from measuring: ‘Your ruler shows they’re equal for this specific rectangle. Can you prove it must be true for every rectangle?’

Converse: worth stating but proof is optional here. If the diagonals of a parallelogram are congruent, then it is a rectangle. The proof uses equal diagonals + parallelogram properties to force all angles to 90° .

Prompt 2 — Rhombus (≈20 min) — Give to all groups once most have Prompt 1

Prompt

Now a different condition: all four sides are equal. Start fresh with a parallelogram where all sides are equal. Mark what you know. Draw both diagonals. What do you notice? Can you prove it?

⚠ What to watch for

Two key observations: (1) diagonals are perpendicular, (2) each diagonal bisects the vertex angles. Both need proof.

Groups trying direct congruence: they will try to prove perpendicularity without going through the isosceles triangles first. They will get stuck. Give the hint below.

Hint if needed: ‘Look at the triangle formed by one diagonal. What kind of triangle is it? What do you know about that kind of triangle from Unit 4?’

The Unit 4 connection to name: each diagonal splits the rhombus into two isosceles triangles — the equal sides of the rhombus are the equal sides of the triangle. The base angles theorem from Lesson 4.3 then gives the base angles equal, which means the diagonal bisects the vertex angle. Once angle bisection is established, SSS on the four small triangles gives perpendicularity via CPCTC + linear pair.

Extension — The Square (give individually when groups have both)

Prompt

You now have results for rectangles and results for rhombi. What diagonal properties must a square have? Do you need to prove anything new, or do the rectangle and rhombus results already give you everything? Write a careful argument explaining why the square’s properties follow by logical inheritance rather than new proof.

Most groups will quickly see that no new proof is needed. The interesting work is articulating the argument precisely: a square is both a rectangle and a rhombus, so it inherits both sets of diagonal properties. Push students to write this as a connected conditional chain using the language from Unit 2.

CONSOLIDATION — THREE PARTS

Part A — Rectangle: Diagonals Are Equal

Walk through the proof. Draw on the board as you go. Use flow chart format to show the two branches ($AB = AB$, $\angle s$ are 90° , $BC = AD$ from 5.2) feeding into SAS, then CPCTC.

Rectangle Diagonal Theorem — Proof

Given: Rectangle ABCD (parallelogram with all right angles). Diagonals AC and BD drawn.

Statement	Justification
ABCD is a rectangle $\rightarrow AB \parallel DC, AD \parallel BC$	Definition of rectangle (a parallelogram with right angles).
$AB = AB$	Reflexive property.
$\angle ABC = \angle BAD = 90^\circ$	Definition of rectangle — all angles right angles.
$BC = AD$	Parallelogram property (Lesson 5.2) — opposite sides congruent.
$\triangle ABC \cong \triangle BAD$	SAS — $BC = AD, \angle ABC = \angle BAD = 90^\circ, AB = AB$.
$AC = BD$ ■	CPCTC — diagonals are equal.
<i>Conditional form: If a parallelogram is a rectangle, then its diagonals are congruent.</i>	

Converse — state but proof is optional

If the diagonals of a parallelogram are congruent, then it is a rectangle. The proof reverses the argument — equal diagonals + parallelogram properties force all four angles to 90° . Useful for coordinate classification.

Part B — Rhombus: Diagonals Perpendicular and Bisect Vertex Angles

Name the Unit 4 connection before beginning the proof. This is the key teaching moment of the lesson.

Name this explicitly — Unit 4 at work

'We just used the base angles theorem from Unit 4 to prove something about a quadrilateral. This is what it means to build a mathematical system: results from earlier proofs become tools for later ones. The base angles theorem was about triangles. The rhombus gives us two isosceles triangles. The theorem transfers.'

Rhombus Diagonal Theorem — Proof

Given: Rhombus ABCD (all four sides equal). Both diagonals drawn, intersecting at E.

Statement	Justification
$AB = AD$	Definition of rhombus — all sides equal.
$\triangle ABD$ is isosceles with $AB = AD$	Definition of isosceles triangle.
$\angle ABE = \angle ADE$	Base angles theorem (Unit 4, Lesson 4.3) — equal sides produce equal base angles.
$AB = AD$, $BE = DE$ (diagonals bisect each other, 5.2), $AE = AE$	Step 1; Lesson 5.2 Property 4; reflexive.
$\triangle ABE \cong \triangle ADE$	SSS.
$\angle AEB = \angle AED$	CPCTC.
$\angle AEB + \angle AED = 180^\circ$	Linear pair — E lies on diagonal BD .
$\angle AEB = \angle AED = 90^\circ$ ■	Two congruent angles in a linear pair must each equal 90° .

Conditional form: If a parallelogram is a rhombus, then its diagonals are perpendicular and bisect the vertex angles.

Second rhombus property: diagonal bisects vertex angles

From Step 3: $\angle ABE = \angle ADE$, so diagonal AC bisects $\angle A$. The same argument with the other diagonal establishes that BD bisects $\angle B$ and $\angle D$. Name this as a second conditional: if a parallelogram is a rhombus, then each diagonal bisects a pair of opposite vertex angles.

Part C — The Square: Inheritance Without New Proof

Write on the board: A square is a rectangle. Pause. A square is a rhombus. Pause. What does it inherit from each?

The argument to give — write it on the board as you speak

A square is a rectangle. Therefore it inherits all rectangle diagonal properties: diagonals bisect each other (parallelogram), diagonals are equal (rectangle theorem).

A square is a rhombus. Therefore it inherits all rhombus diagonal properties: diagonals are perpendicular, diagonals bisect vertex angles (rhombus theorem).

No new proof is required for any of these. The square sits at the bottom of the parallelogram branch because it satisfies the most constraints — and therefore inherits the most properties.

Then ask: 'Is there any property a square has that neither the rectangle nor the rhombus gives you?' The answer is no — and that is the payoff. Understanding the hierarchy means never needing to prove what you can inherit.

SHARE-AND-REVISE NOTES (≈12 MIN)

Students remain in groups. Each student writes their own responses to the three note sheet prompts individually — first the rectangle conditional and proof, then the rhombus, then the square inheritance argument. Give 10 minutes. Then ask one group to share each section. Students revise their own notes based on what they hear.

How to run this

Goal: not one correct answer but students hearing multiple valid formulations and choosing the clearest.

After sharing: ask ‘does anyone want to suggest a tighter statement? A cleaner proof?’

Do not: show the teacher reference proofs before students have written their own versions. The sharing step is not optional. Students who skip individual writing and copy from a group member miss the metacognitive work.

STUDENTS MAKE NOTES ON

Students make notes on:

- Rectangle diagonal theorem: additional constraint (all right angles), conditional form, proof summary (SAS $\rightarrow$ CPCTC)
- Rhombus diagonal theorem: additional constraint (all sides equal), two conditional forms (perpendicular; bisects vertex angles), proof summary (base angles theorem $\rightarrow$ SSS $\rightarrow$ CPCTC)
- Square: why both sets of diagonal properties follow without new proof — written as a conditional chain
- Properties summary table (students complete their own version)

Narrative Arc — End of consolidation — transition to Lesson 5.4

“You now have diagonal properties for rectangles, rhombi, and squares — all proven, none memorized. Next lesson we leave the parallelogram family and look at the shapes that live outside it: trapezoids and kites. The same strategy applies: find the hidden triangles, prove congruence, extract by CPCTC. But the auxiliary constructions will be different. Watch for where the symmetry lives.”

Connection Points

- Draws from Lesson 4.3: base angles theorem is the essential tool in the rhombus proof — name this explicitly
- Draws from Lesson 5.2: parallelogram Properties 1 (opposite sides) and 4 (diagonals bisect) are cited in both proofs
- Draws from Lesson 5.1: the hierarchy explains why the square proof requires no new work
- Feeds into Lesson 5.5: perpendicular diagonals (rhombus/square) and equal diagonals (rectangle/square) are key coordinate classification tools