Lesson 4.3 — Teacher Reference Notes

The Base Angles Theorem and Its Converse

Unit 4: Triangle Theorems • Day 3 of 6

At a Glance	
	Pacing: Thinking task: Two Hikers (~15 min) • Extension 1: proof (~20 min) • Extension 2: converse (~15 min) • Gallery walk if time allows • Consolidation: proof + converse + biconditional (~25 min) • Notes + CYU begins (~15 min) **Materials:** Boards and markers • Paper for students who want to draw diagrams **Nav instrument rows active:** Row 3 (Base Angles Theorem) activates today — all three levels **Task delivery:** The thinking task is read aloud — not written on the board. Extensions are given individually to groups as they are ready.

OBJECTIVES

By the end of this lesson students can:

- State the base angles theorem and its converse as conditional statements.
- Prove the base angles theorem using the angle bisector and SAS.
- Explain why the converse requires a separate proof and why accepting both forms a biconditional.
- Use both the theorem and converse to find missing sides and angles and solve algebraic problems.

Narrative Arc — Open class — read the task aloud

"Two hikers begin at the same point. They walk at the same steady pace but in different directions — not opposite each other. After walking for the same amount of time, they both stop and turn to face each other directly. Draw this situation. What do you notice? Make as many conjectures as you can. Test them on different starting directions. Which of your conjectures can you prove? Which ones are you just observing?"

TASKS

Thinking Task — Two Hikers (≈15 min)

Read aloud. Do not write it on the board. No further instructions before groups begin.

Aria and Leo begin at the same point in a wide-open field. They walk at the same steady pace but in different directions — not opposite each other. After walking for the same amount of time, they both stop and turn to face each other directly.

- Draw this situation. What do you notice?

- Make as many conjectures as you can. Test them on different starting directions.
- Which of your conjectures can you prove? Which ones are you just observing?

What to watch for

The geometry: The equal walking distances are the equal sides of an isosceles triangle. The line between the hikers is the base. Students typically notice: both hikers are always the same distance from the starting point; a triangle is always formed (even though the task never says so); the angles they turn through to face each other appear to be equal.

A reasoning path worth celebrating if it appears: When the hikers turn to face each other, they create exterior angles at the two base vertices. By the symmetry of the equal walking distances, these exterior angles are equal. The interior base angles are supplementary to their respective exterior angles. Since both exterior angles are equal, both expressions ‘180° − exterior angle’ are equal by Common Notion 1. Therefore the interior base angles are equal. This is a valid independent argument — celebrate it explicitly.

Do not confirm or name the conjecture. When groups are convinced the base angles are equal, give them Extension 1.

Extension 1 — Prove It (given individually when groups have the conjecture)

Prompt

Given: Triangle ABC with AB = AC. Prove: ∠ABC ≅ ∠ACB. Write a complete proof. State every step and cite every reason. If you are stuck, think about how you might turn one triangle into two.

Teacher moves

Most groups: will attempt symmetry or folding arguments. Good intuitions — push them toward deductive steps: ‘How do you state that as a justification you could cite?’

Key insight: draw the angle bisector from A to BC. This creates two triangles that can be compared by SAS. Angle bisector from A is Proposition 9 — established in Unit 3.

Hint if needed: *‘Is there a construction from earlier in the course that could split this triangle into two smaller ones?’*

Extension 2 — The Converse (given individually after Extension 1)

Prompt

You proved: if two sides of a triangle are equal, then the angles opposite them are equal. What about the reverse? If two angles of a triangle are equal, must the sides

opposite them also be equal? Is this the same claim you just proved, or a different one? Can you prove it? Or can you find a reason to doubt it?

What to expect

Dead end: Groups who try to apply the angle bisector approach will find it does not transfer directly — the symmetry argument relied on knowing the sides were equal, which is exactly what the converse is trying to establish. This dead end is productive: it is the felt experience of why a converse needs its own proof.

Correct intuition: Groups will likely conjecture the converse is true. That is correct — but it does not follow from logic alone that a converse must be true because the original is. The consolidation addresses this directly.

CONSOLIDATION — THREE PARTS

Part A — The Proof (walk through together)

Work through the proof together. Name the angle bisector as the key move and explain why it is available — Proposition 9, established in Unit 3.

Base Angles Theorem (Euclid Proposition 5) — Proof

Given: Triangle ABC with AB = AC. Construction: Draw the angle bisector from A to BC. Label where it meets BC as point D. (Proposition 9 — established in Unit 3.) Prove: ∠ABC ≅ ∠ACB.

Statement	Justification
AB = AC	*Given.*
∠BAD ≅ ∠CAD	*Definition of angle bisector.*
AD = AD	*Reflexive property.*
△ABD ≅ △ACD	*SAS.*
∠ABC ≅ ∠ACB ■	*CPCTC.*

Conditional form: If two sides of a triangle are equal, then the angles opposite those sides are equal.

Why this proof is cleaner than Euclid's original

The angle bisector (Proposition 9) is the tool that makes this proof clean. Euclid could not use it at Proposition 5 — he had not yet established it. His original proof extends the equal sides beyond the base and uses SAS twice in a more involved auxiliary construction. It is valid but considerably harder to follow.

Our proof is more direct because we built and accepted the tools in a different order. The elegance of a proof depends partly on what has been established before it.

Part B — The Converse (acknowledge and accept)

Acknowledge what groups found when they attempted the converse: the intuition is correct, but the proof requires reasoning by contradiction — a technique not yet developed in this course. Accept the converse today.

Optional extension: the proof by contradiction is included in the reference below for teachers who want to share it with groups that are ready.

Optional Extension — Proof of the Converse by Contradiction (Euclid Proposition 6)

Given: Triangle ABC with ∠ABC ≅ ∠ACB. Prove: AB = AC.

Assume for contradiction that AB ≠ AC. Suppose AB > AC. By Proposition 3, cut off AD from AB equal to AC, with D between A and B.

Now △DBC and △ACB share base BC. DC = AC by construction. ∠DBC = ∠ACB (given). So △DBC ≅ △ACB by SAS.

But △DBC is a proper part of △ACB — it cannot be congruent to the whole. This contradicts Common Notion 5: the whole is greater than the part.

Therefore AB = AC. ■

When sharing this proof: guide students to articulate precisely where the contradiction arises — the congruence of a triangle with a proper subset of itself violates Common Notion 5.

Part C — The Biconditional (name it explicitly)

Say directly: ‘We proved the theorem. We accepted the converse. In Unit 2 we established that a conditional and its converse are independent claims — one can be true while the other is false. Here both happen to be true.’

When both a conditional and its converse are true, together they form a biconditional:

A triangle has two equal sides if and only if it has two equal base angles.

The biconditional lets us move freely in either direction: equal sides imply equal angles, and equal angles imply equal sides. Students will use this throughout the rest of the unit.

Connect to Unit 2

Students established in Unit 2 that a biconditional requires both directions to be proven (or accepted) independently. The fact that a theorem is true does not guarantee its converse is true — that is a fact about this particular theorem, not a logical necessity. Name this explicitly. Students who remember Unit 2 should recognize the structure immediately.

DEFINITIONS REFERENCE — TEACHER COPY

Isosceles Triangle

A triangle with at least two equal sides. The two equal sides are called legs. The third side is called the base. The angle between the two legs is the apex angle. The two angles at the base are the base angles.

Base Angles Theorem (Proposition 5)

If two sides of a triangle are equal, then the angles opposite those sides are equal.

Conditional form: If a triangle is isosceles, then its base angles are congruent.

Converse of the Base Angles Theorem (Proposition 6)

If two angles of a triangle are equal, then the sides opposite those angles are equal.

Conditional form: If a triangle has two congruent angles, then the sides opposite those angles are congruent.

Biconditional (both together)

A triangle has two equal sides if and only if it has two equal base angles.

This is the biconditional: both the theorem and its converse are true, so they can be stated as a single if-and-only-if claim. Either condition guarantees the other.

STUDENTS MAKE NOTES ON

✎ **Students make notes on:**

- Isosceles triangle vocabulary: legs, base, apex angle, base angles
- Base angles theorem: conditional form, diagram, proof (the five-step SAS proof with angle bisector)
- Converse: conditional form, diagram, how established (“accepted without proof” or proof by contradiction if covered)
- The biconditional: written as an if-and-only-if statement, with the Unit 2 connection named

▶ **Narrative Arc** — End of consolidation — connect to the arc of the unit

“We have now proven four things from the axioms: the triangle angle sum, the exterior angle theorem, the polygon angle sum formula, and the base angles theorem. Every one of those required a construction to make the proof possible. In the next lesson we ask: what do the constructions themselves guarantee? What does it mean to lie on a bisector?”

Connection Points	• Draws from Unit 3: the angle bisector (Proposition 9) is the key tool in the proof; CPCTC and SAS are the same tools used in every Unit 3 proof • Draws from Unit 2: the biconditional structure and the independence of a conditional and its converse are directly from Unit 2 • Draws from Lesson 4.2: the altitude proof established that the altitude from the apex of an isosceles triangle is the perpendicular bisector of the base — CPCTC gives PH = QH, so H is the midpoint • Feeds into Lesson 4.4: the angle bisector theorem and perpendicular bisector theorem build on the same SAS + CPCTC structure used today