Lesson 7.2 — Teacher Reference Notes

If We Can Measure the Angle, We Can Measure Anything

Unit 7: Trigonometry • Days 2–3 of unit • Day 1: Fluency + Device Design • Day 2: Field Measurement

At a Glance	
	Day 1 pacing: Part I: fluency at boards with table-first estimation (~30 min) • Part II: design challenge (~45 min) • Do not feel obliged to complete all practice problems — if students have it, move to the design challenge **Day 2 pacing:** Precision address (~5 min) • Field measurement (~45–60 min) • Return: calculations + percent error (~20–30 min) **Materials Day 1:** Boards and markers • Calculators • Class trig table (still posted) • Design materials: protractors, string, washers, tape, cardboard, rulers, straws **Materials Day 2:** Completed devices • Measuring tapes • Field measurement sheets • Calculators **Nav instrument rows active:** Row 2 (Solving for Missing Sides) activates today — all three levels **Table-first rule:** Before touching the calculator for any problem, groups must consult the class trig table and record a written estimate. This is not optional — it reinforces that trig values are predictable quantities, not mysterious calculator outputs.

OBJECTIVES

By the end of this lesson students can:

- Select the correct ratio (sin, cos, or tan) to solve a right triangle given different starting information.
- Estimate trig values from the class table before computing exactly.
- Design and build a device capable of measuring an angle of elevation, with attention to sources of precision error.
- Use the device to measure inaccessible heights using right triangle trigonometry.
- Calculate percent error and reason about how angle measurement error propagates into height estimation error.

Narrative Arc — Open Day 1 — before Part I

> *"In Lesson 7.1 you discovered that one acute angle determines all the ratios in a right triangle. Today you use that. If you know one angle and one side, you know everything. And that means: if you can measure one angle and one distance, you can measure anything."*

DAY 1 — PART I: FLUENCY AT THE BOARDS (≈30 MIN)

Groups work at boards. The class trig table must still be posted and visible. Before any calculator use, every group records a written estimate from the table.

The table-first habit — enforce consistently

'Before you touch the calculator, look at the class table. Between which two angles in your table is 35°? What does that tell you the sine should be approximately? Write your estimate first. Then compute exactly. How close were you?'

This habit matters because it builds number sense for trig values. Students who only ever use a calculator have no feel for whether sin(80°) = 0.98 is reasonable or whether they accidentally computed cos(80°). The estimate is the check.

Practice problems and additional problems — teacher reference:

#	Given	Find	Answer + ratio used
P1	35°, hyp = 20 cm	Opposite side	opp = 20·sin(35°) ≈ 11.47 cm. Table estimate: sin(35°) ≈ 0.57 (between 30° and 50° entries). Exact: ≈11.47.
P2	52°, adj = 9 m	Hypotenuse	hyp = 9/cos(52°) ≈ 14.63 m. Note: cos goes on the bottom because cosθ = adj/hyp, so hyp = adj/cosθ.
P3	Vary — see note	All sides and angles	Select triangle with one angle and one side. Students should find: second angle = 90° − θ; other two sides using trig or Pythagorean theorem.
E1	41°, hyp = 12 m	All sides and angles	opp = 12·sin(41°) ≈ 7.87, adj = 12·cos(41°) ≈ 9.06. Second angle = 49°.
E2	63°, adj = 7 cm	All sides and angles	hyp = 7/cos(63°) ≈ 15.43, opp = 7·tan(63°) ≈ 13.74. Second angle = 27°.
E3	28°, opp = 9 ft	All sides and angles	hyp = 9/sin(28°) ≈ 19.16, adj = 9/tan(28°) ≈ 16.91. Second angle = 62°.
E4	52°, adj = 15 m	All sides and angles	hyp = 15/cos(52°) ≈ 24.38, opp = 15·tan(52°) ≈ 19.20. Second angle = 38°.
E5	37°, hyp = 20 ft	All sides and angles	opp = 20·sin(37°) ≈ 12.04, adj = 20·cos(37°) ≈ 15.97. Second angle = 53°.

Multiple solution path note — name this explicitly

Encourage two paths for each complete-the-triangle problem: (1) find one side with trig, then use the Pythagorean theorem for the second; (2) find the other acute angle using the triangle angle sum theorem (90° + θ + comp = 180°), then use a different ratio. When both paths give the same answer, students feel the consistency of the system.

DAY 1 — PART II: DESIGN CHALLENGE (≈45 MIN)

Launch prompt — display on board

If trig ratios depend on angles, and if we can measure one angle and one side, then we can measure anything. Each group is to design a way to measure angles of elevation. A reward will be given to the group whose design leads to the most accurate measurements on Day 2. Your device must be able to measure an angle of elevation to a distant object. Think carefully about precision. What will introduce

error? How can you reduce it? Be prepared to explain how your device works and how you will use it in the field.

Materials to have available

Protractors • String or fishing line • Washers or weights • Tape • Cardboard / poster board • Rulers • Straws • If a group has a reasonable request for a reasonable design, accommodate it.

Precision coaching — questions to use as you circulate

- How will you hold it steady while measuring?
- How will you ensure the string or weighted indicator hangs freely?
- How will you measure the horizontal distance accurately?
- How will you account for eye height?
- What happens to your measurement if you are off by 1°? Off by 5°?

It may be helpful to mark a known height and distance in the room, for students to test their devices on, and create a better product through feedback.

⚠️ Year 1 reflection — two very different outcomes

Period 4 (good): almost all groups built clinometers. Most could use the device and set up the diagram. Some creative alternatives included gravity-free devices and rotating rulers along drawn protractors.

Period 6 (disappointing): many devices lacked precision — bases not held parallel to the floor, only every 10° marked, thick rulers instead of string. Many students had not thought through how to actually use the device and were lost in the field. Students rushed the project sheet rather than engaging with it.

The difference was ownership. Period 6 treated this as a task to complete. Address this explicitly at the end of Day 1 and the start of Day 2: your device is a scientific instrument. Precision matters. Your grade is not based on completion — it is based on accuracy.

DAY 2 — FIELD MEASUREMENT

Open Day 2 with this — say it before releasing groups to the field

'Your device is a scientific instrument. Scientists don't get partial credit for being approximately right about the boiling point of a compound. Engineers don't build bridges that are roughly the right length. Precision matters today. Take your time. Read the angle carefully. Measure the horizontal distance accurately. Account for eye height.'

Pre-select measurement objects

Choose 2–3 objects in advance whose actual heights you know or can estimate. Indoor objects avoid weather constraints and allow easy verification:

- Gym ceiling height

- Top of bleachers
- Top of a stairwell or mezzanine
- A specific point on a tall wall

Knowing the true height allows you to compute actual percent error for each group, which makes the back-of-sheet questions more meaningful.

The percent error question — defend giving it

The back of the field measurement sheet asks: if your angle was 1° smaller, what height would you estimate? What percent error does a 1° angle error cause? Would a 1° error always cause the same percent error?

This question is worth defending. Better to give students time to struggle with it independently before providing hints. The key insight: the relationship between angle error and height error is non-linear (it depends on the tangent function, which is not linear). A 1° error near 0° produces a very different height error than a 1° error near 85°.

If a group notices this: celebrate it explicitly. They have discovered that trig functions are not linear, and that measurement error analysis requires calculus (the derivative of tanθ) to handle properly. That is a genuinely advanced mathematical observation.

CYU 7.2 — ANSWER KEY

Q	Answer	Teacher note
1	opp = 12·sin(41°) ≈ 7.87 m, adj = 12·cos(41°) ≈ 9.06 m.	Straightforward. Watch for sin/cos confusion.
2	hyp = 7/cos(63°) ≈ 15.43 cm, opp = 7·tan(63°) ≈ 13.74 cm.	Common error: multiplying by cos instead of dividing.
3	hyp = 9/sin(28°) ≈ 19.16 ft, adj = 9/tan(28°) ≈ 16.91 ft.	Can also find adj with Pythagorean theorem after finding hyp.
4	h = 18·tan(35°) ≈ 12.61 m.	Diagram is essential — the shadow is adjacent, height is opposite.
5	h = 120·tan(22°) ≈ 48.50 m.	Same structure as Q4. Distance horizontal = adjacent.
6	h = 3000·tan(6°) ≈ 315.6 m. (3 km = 3000 m)	Unit conversion needed. Accept 0.316 km.
7	Horizontal = 80/tan(14°) ≈ 321.1 m. Straight-line = 80/sin(14°) ≈ 330.7 m.	Angle of depression: equal to the angle of elevation from the boat. The two right triangles are identical.
8	Rise = 12·sin(5°) ≈ 1.046 m. Run = 12·cos(5°) ≈ 11.95 m. Ratio 1.046/11.95 ≈ 1:11.4, steeper than 1:12. Not ADA compliant.	The ADA question is the richest part. Push students to interpret 1:12 as a slope ratio, not a percentage.

STUDENTS MAKE NOTES ON

✎ Students make notes on:

- The ratio selection strategy: which ratio to use given which sides are known or wanted (see student note sheet)
- Worked example: at least one complete triangle problem from the boards, showing the table estimate and the calculator solution
- Field measurement setup: a diagram showing angle of elevation, horizontal distance (adjacent), height (opposite), and the proportion used

Connection Points	• Draws from Lesson 7.1: the class trig table is the estimation tool; SOH-CAH-TOA is the selection framework • Draws from Unit 6: indirect measurement using similar triangles (Lesson 6.4) is the conceptual predecessor; today's method is more precise because it uses a specific angle rather than a similar triangle setup • Draws from Unit 4: triangle angle sum theorem is used to find the second acute angle in complete-the-triangle problems • Feeds into Lesson 7.3: the percent error question plants the seed that angle-to-value is not the only direction — value-to-angle is the inverse trig problem