

Unit Zero

Learning How to Think Together

4 lessons • No assessment • Gallery walk consolidation throughout

Before we begin any formal curriculum, we begin by learning how to be in this class.

Unit Zero exists outside the traditional scope of content. No theorems are proven here, no vocabulary introduced, no formulas established. The purpose of this unit is to build the intellectual and social norms that will make meaningful mathematical work possible for the rest of the year — and to give students their first sustained experience of what thinking mathematically actually feels like from the inside.

Most students arrive in a geometry class having spent years learning how to succeed at school without necessarily learning much. They know how to wait for direction, copy what the teacher writes, follow procedures they do not understand, and produce answers that look right without being able to say why. These habits are rational responses to the classrooms they have been in. They are not character flaws. But they are habits that will prevent learning in this classroom, and Unit Zero is the structured disruption that begins to replace them.

The disruption happens through experience, not explanation. From the first day, students work in visibly randomized groups of three at vertical non-permanent surfaces. They are given problems that cannot be solved by remembering a procedure, because no procedure exists yet. They are asked to try things, test them, argue about them, and revise them. Mistakes are treated as information. Partial ideas are treated as contributions. The classroom communicates, through its structure, what kind of thinking is expected here — and most students receive that communication more readily than they would receive an explanation of it.

One of the clearest confirmations of this came in the first year I taught the unit seriously. My teaching partner and I had prepared several days of explicit norm-setting activities — discussions, agreements, expectation-setting conversations. By the third day, we realized we did not need them. The norms had already been established. Students were collaborating, managing disagreement, and taking ownership of their thinking. The tasks had done the work that our planned discussions were supposed to do.

The Four Tasks and Their Progression

The tasks in this unit come from two sources. The Warehouse Discount Problem and the Palindromes task are drawn from Mason, Burton, and Stacey's *Thinking Mathematically* — a book worth working through yourself before teaching it to students.

The mathematical thinking framework used throughout the unit is adapted from that book as well.

Thinking Mathematically (2nd edition) — Mason, Burton, and Stacey (Pearson, 2010)

The source for Lessons 0.1 and 0.2, and the intellectual foundation for the mathematical thinking framework used across all four lessons. The framework — specializing, organizing, generalizing, conjecturing, justifying — is adapted from it. Doing the tasks yourself, before teaching them, is strongly recommended.

The Chessboard Paths task in Lesson 0.3 comes from James Tanton's Teaching Company Geometry course, where Tanton shares the story of his own encounter with the puzzle as a student. The visual sum tasks in Lesson 0.4 are drawn from the broader mathematical folklore of visual proof, with the consecutive odd numbers argument appearing in various forms across many sources.

All four tasks are sequenced deliberately.

Lesson 0.1 introduces specializing and testing — the habit of trying specific examples before claiming anything general, and the habit of looking for counterexamples rather than only confirming evidence. The Warehouse Discount Problem surfaces confirmation bias in a low-stakes setting. Guess My Rule makes the cost of confirmation bias vivid in a way that students remember.

Lesson 0.2 adds organizing and generalizing — the recognition that how you arrange your examples matters, and that moving to variables is often the key step toward showing something is always true rather than just usually true. The Palindromes task naturally rewards organization and invites algebraic generalization.

Lesson 0.3 pushes toward convincing arguments — the distinction between showing that something appears to be true and showing that it must be true. The Chessboard Paths task makes this distinction unavoidable: to show a path exists, one example is enough. To show no path can exist, an argument is required. This is the first time in the course that students feel the difference from the inside, and it is a feeling worth naming and returning to.

Lesson 0.4 introduces visual reasoning as a tool that supports all of the above. The sum tasks in this lesson have visual proofs that are more elegant and more convincing than algebraic ones — a reminder that mathematical argument takes many forms, and that the goal is always understanding, not just symbolic manipulation.

The Mathematical Thinking Framework

Across all four lessons, students are developing habits that Mason, Burton, and Stacey describe as the traits of mathematical thinking. These are not steps to follow in order. They are dispositions to cultivate over time, and students will move between them recursively throughout the year.

Get started: Try something. Draw, guess, rephrase. You do not need to know where you are going to begin.

Specialize: Try specific examples. Random cases are fine to start; systematic cases help you notice patterns.

Organize: Look for structure in your cases. How you arrange what you have found often reveals what you have not yet seen.

Generalize: Ask what might always be true. Introducing variables is usually the move that turns a pattern into a claim.

Test: Try to break your conjecture. One counterexample disproves a claim. A thousand confirming examples do not prove it.

Conjecture: State what you believe to be true, clearly enough that someone else could test it.

Justify / Convince: Explain why it must be true. The standard of convincing rises across the unit: convince yourself, then your group, then your peers, then the world.

Reflect: Write about what you tried, what you learned, what questions you still have, and what you would do differently.

Post this framework somewhere permanent in your classroom. Do not explain it on day one — let students encounter it through the tasks first. By Lesson 0.3, when you give them the list explicitly, they will recognize it as a description of what they have already been doing.

What This Unit Is Not

Unit Zero has no Check Your Understanding problems, no meaningful notes, and no navigation instrument. Students are not assessed on any of this content. There is nothing to memorize and no rubric to self-evaluate against.

Consolidation in every lesson takes the form of a gallery walk, because these tasks are open and divergent — different groups will have explored different directions, and sharing that variety is more valuable than converging on a single correct answer. The gallery walk is also a cultural tool: it teaches students that other groups' thinking is worth examining, that partial progress is worth sharing, and that the room is a resource.

What is being built here cannot be assessed by a test. It is a set of habits, a classroom culture, and the beginning of an intellectual disposition that will take the whole year to develop. Treat it accordingly.

A note on downloadable materials: Unit Zero is the only unit in this book with no student-facing downloads. There are no student note sheets, no Check Your Understanding problems, no homework sets, and no navigation instrument. The only download for each lesson is a Teacher Reference Notes document containing the task prompt, extension ladder, hints, and full consolidation guidance including detailed proofs. Starting in Unit One, each lesson includes the full set of downloadable materials.