

Unit 2

Logic and Reasoning

9 lessons across 11 days • 1 mid-unit quiz • CYU replaces homework throughout

Unit Navigation Instrument

Big Idea: Reasoning is only as reliable as its structure and its starting points — this unit builds the tools to evaluate both.

Subtopic	Basic	Intermediate	Advanced
Inductive Reasoning → <i>Recognizing patterns and their limits</i>	Define inductive reasoning and complete a sequence.	Write the n th term of a sequence; name the type of reasoning used.	Explain the difference between a true conclusion and a guaranteed one; provide a counterexample to a plausible conjecture.
Deductive Reasoning → <i>Building conclusions that must follow</i>	Define deductive reasoning.	Contrast inductive and deductive reasoning with specific examples.	Explain why careful, logically correct thinkers can still reach different conclusions.
Conditional Statements → <i>The grammar of logical argument</i>	Write all four forms of a conditional statement; identify truth values.	Provide counterexamples for false forms; identify equivalence pairs; determine whether a statement is biconditional.	Explain why the equivalences hold using a Venn diagram; rewrite a statement as a biconditional and justify the decision.
Evaluating Arguments → <i>Distinguishing structure from content</i>	Identify arguments as valid, sound, or invalid.	Name the fallacy in each invalid argument; explain in one sentence why the conclusion does not follow.	Use a Venn diagram to show why a fallacy fails; explain why denying the consequent is valid while affirming the consequent is not.
Named Argument Forms → <i>Naming the tools of deduction</i>	Match each argument to its name (law of detachment, law of syllogism, denying the consequent, disjunctive syllogism).	Apply denying the consequent and the law of syllogism to draw valid conclusions; write a chain symbolically.	Solve a multi-step Carroll-style puzzle using the law of syllogism and contrapositive together.

Transversal Angle Relationships → <i>Using deductive reasoning for geometry</i>	Name corresponding, alternate interior, alternate exterior, and co-interior angles.	Use angle relationships to find missing angle measures; solve algebraic problems involving parallel lines cut by a transversal.	Given one angle relationship as a premise, construct a deductive argument for why the others must hold — using only vertical angles and linear pairs.
Axioms and Shared Premises → <i>Where deductive systems begin</i>	Define axiom; state one postulate and one common notion from Euclid's system.	Explain why axioms are needed and why they cannot be proven.	Explain how careful, logically correct thinkers can disagree without either making an error — and connect this to the role of shared premises.

The downloadable versions of this instrument include problem numbers in each cell, aligned to the CYU sets for each lesson. As each lesson introduces new content, new rows become active in the CYU. Earlier rows continue to appear as spiral problems throughout the unit, providing the spaced retrieval practice that builds durable understanding.

A note on differentiation days: this unit does not include a scheduled differentiation day. The volume of content and the mid-unit quiz serve the resetting and consolidating function. If your students show signs of struggling across multiple rows after Lesson 2.3, consider inserting a differentiation day before the mid-unit quiz. Use your navigation instrument data to make that call.

The Story This Unit Is Telling

This unit has an arc, and the arc matters. It is worth naming at the start — for yourself as you plan, and eventually for students as they move through it.

We begin with inductive reasoning: the tool humans use most naturally, and the one that fails most visibly. Students have been using inductive reasoning since Unit Zero — noticing patterns, making conjectures, testing them. In Lesson 2.1, we make that process explicit, give it a name, and then watch it fail. The circle task is designed specifically for this: a pattern that holds for five cases and breaks on the sixth. The failure is not incidental. It is the point.

The failure of inductive reasoning motivates the question that drives the rest of the unit: if we cannot trust patterns, what can we trust? The answer is deductive reasoning —

argument structures that guarantee their conclusions when the premises are true. Lessons 2.2a and 2.2b give students the primary tool of deductive reasoning: the conditional statement and its four forms. The mid-unit quiz closes that chapter.

But then a harder question arrives. If deductive reasoning is so reliable, why do careful, well-intentioned people still disagree about important things? Lesson 2.3 builds toward that question by teaching students to evaluate the structure of arguments. Lesson 2.4 names the specific valid argument forms and their fallacious counterparts. We can now revisit the question with a twist: if two people use correct argument structures and avoid the fallacies, how could they still disagree? When this question has been posed in class, a student has always eventually offered some version of the answer: they must have started from different premises.

That insight is the pivot on which the entire unit turns — and it connects directly to contested arguments in the real world. When people disagree about deeply held beliefs, the problem is rarely that one side is reasoning incorrectly. More often, both sides are reasoning from different starting premises. Productive debate requires establishing shared premises first. Euclid understood this. Lesson 2.6 names this structural answer: an axiomatic system. Shared starting premises — self-evident truths accepted without proof because they are too fundamental to derive from anything simpler. Ten of them, in Euclid's case, from which 465 other results follow necessarily.

The conditional statement is the grammatical unit of every geometric proof that follows. The law of syllogism is the mechanism that makes every proof chain possible. The axiomatic system is the foundation that makes certainty achievable. Students who leave this unit understanding those three things are ready for proof — and better equipped to engage in meaningful argument beyond mathematics.

What Changed From Year 1 and Why

A note for the reader: this unit was substantially rewritten after the first year of teaching it. The account that follows is honest about what did not work and why. This is what improving as a teacher actually looks like — you teach something, you learn from the results, you revise, and the next version is better. Teachers who adopt these materials should expect to do the same. The course is a starting place, not a destination.

The original unit was taught in nine days with daily quizzes that cost consolidation time. Looking back, two structural problems were clear.

The first was that the original Lesson 2.2 was doing too much. Deductive reasoning, conditional statements, converse, inverse, contrapositive, biconditionals, and Venn diagrams were all in one lesson. That is at minimum two lessons' worth of content. The

solution was to split it: 2.2a covers deductive reasoning and conditional statements together — the transition from ‘inductive reasoning can fail’ to ‘what do we do instead’ is one coherent idea. Lesson 2.2b gives the four forms and biconditionals their own lesson with a dedicated thinking task.

The second problem was uneven cognitive load. The named argument forms — law of detachment, law of syllogism, disjunctive syllogism, denying the consequent — arrived on top of students who were already carrying conditionals, biconditionals, and the valid/sound distinction. By the time the argument laws showed up, working memory was full. The solution was to give them their own lesson: 2.4 is entirely dedicated to the named forms, with a thinking task that leads students to discover the law of syllogism before it is named. The Lewis Carroll puzzle, which appeared as an extension in the original 2.2, is now the culminating challenge of 2.4 — where it belongs, because it requires both the law of syllogism and the contrapositive.

Lesson 2.5 on transversal angle relationships is new to this sequence. It was originally part of the axioms lesson but deserves its own day: it is the first time in the course that students apply logical reasoning directly to geometric figures, and that transition is significant enough to give space. Students conjecture the angle relationships inductively, then derive them deductively from one accepted premise using tools they already have. The Euclid’s Handbook connections are noted in the lesson for teachers who are tracking propositions.

A note on homework: in this unit, the CYU problems serve as both in-class practice and take-home work. There are no separate homework sets. The CYU problems are tiered by the navigation instrument rows covered to that point, with spiral problems returning to earlier content throughout.

Lesson Sequence Overview

Lesson 2.1: Inductive Reasoning — three tasks, the epistemological gut-punch, the distinction between pattern and explanation

Lesson 2.2a: Deductive Reasoning and Conditional Statements — skyscraper puzzle, lunch party, guided notes on the conditional

Lesson 2.2b: Converse, Inverse, Contrapositive, Biconditionals — discovery task, Venn diagram equivalence, inverse error named

Mid-unit quiz: covers rows 1–3 of the navigation instrument

Lesson 2.3 Day 1: Always/Sometimes/Never, argument sorting, valid/sound/invalid distinction, CYU begin in class

Lesson 2.3 Day 2: Monty Python activator, naming the fallacies, connection to conditional forms, full notes and CYU

Lesson 2.4 Day 1: Named argument forms — chain task, law of syllogism discovered, all four valid forms named, notes, CYU begin in class

Lesson 2.4 Day 2: Carroll puzzle resolved, valid vs. fallacy comparison table, full CYU

Lesson 2.5: Transversal Angle Relationships — first deductive geometry, conjecture then derive

Lesson 2.6: Axioms and Shared Premises — Sudoku task, the disagreement question, Euclid's 10 axioms