Unit 8: Circles: Self-Evaluation Rubric: Where I am, where I am going.

Big Idea: *A circle is defined by a single constraint, every point is the same distance from the center, and nearly everything about circle geometry is a logical consequence of that one fact combined with tools we already have.*

Subtopic	Basic	Intermediate	Advanced
What is a circle? *→ Defining the object*	Can state the definition of a circle in terms of equal distance from a center, and use it to explain why all radii must be equal.	Can derive the equation $(x - h)^2 + (y - k)^2 = r^2$ by applying the distance formula to the definition, and identify the center and radius from an equation in standard form.	Can explain why the unit circle is a special case of this definition and use the equation to find missing information. For example, reasoning about what equation fits a circle passing through given points.
All circles are similar *→ Why proportional reasoning works*	Can explain that any circle can be mapped onto any other by a dilation, and that this makes all circles 'the same shape at different sizes' — unlike squares or triangles, where shape can still vary.	Can connect this to the Unit 6 definition of similarity: angles are preserved (always 360°) and lengths scale by a constant factor, and use this to explain why arc length and sector area are proportional to the central angle.	Can explain why circles are unique in this property: because the definition contains only one free parameter (the radius), once the radius is fixed the shape is completely determined — there is no way for two circles to have the same size but different shapes.
Arc length and sector area *→ Applying proportional reasoning*	Can identify an arc and a sector and explain the relationship between a central angle and its arc measure.	Can set up and solve arc length and sector area problems by treating the central angle as a fraction of 360°.	Can work backwards from arc length or sector area to find a missing angle or radius, and can explain why the proportional method works rather than just applying it.
Central and inscribed angles *→ The spine of the unit*	Can state the inscribed angle theorem — an inscribed angle equals half its intercepted arc — and apply it to find a missing angle when the arc is given.	Can explain why the inscribed angle theorem is true using the isosceles triangle argument from the center, and can apply it correctly in all three cases: when the center is inside, on, or outside the inscribed angle.	Can use the inscribed angle theorem as a tool to prove other results — including Thales' theorem (an angle inscribed in a semicircle must be 90°) and the fact that opposite angles of an inscribed quadrilateral are supplementary.
Tangent lines and chord theorems *→ Consequences of the definition*	Can state that a radius drawn to a point of tangency is perpendicular to the tangent line, and apply this to find missing lengths or angles.	Can explain why the radius-tangent relationship must be true using a shortest-distance argument, and can apply the perpendicular-bisector chord theorem to solve problems.	Can connect both theorems to prior units — recognizing the chord bisector as an application of Unit 3's perpendicular bisector work, and the tangent theorem as a direct consequence of the definition of a circle.

Subtopic	Basic	Intermediate	Advanced
Circles and triangles: closing the loop *→ Synthesis with Units 6 and 7*	Can identify the chord-sine relationship and use the formula Area = ½ab · sin C to find the area of a triangle given two sides and an included angle.	Can explain how the Law of Sines from Unit 7 is a circle theorem in disguise — that the ratio a/sin A equals the diameter of the circumscribed circle.	Can bring together circle theorems, triangle congruence, and trigonometric tools in multi-step problems, recognizing which tool is appropriate and explaining why.

What This Unit Is About

Most students arrive at a circles unit having learned, somewhere, that the equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$, that the area is πr^2, that the circumference is $2\pi r$, and that there are some angle relationships involving inscribed angles that they used to know but cannot quite reconstruct. They know circles the way they know most school mathematics: as a collection of facts, each of which has to be remembered separately, and any of which can be forgotten without the others noticing.

This unit is a different proposition. Every major result in it: the equation of a circle, the universality of π, the arc length and sector area formulas, the inscribed angle theorem and its corollaries, the chord-bisector theorem, the radius-tangent theorem, and the Law of Sines reinterpreted as a circle theorem, follows from a single fact: the definition of a circle as the set of all points equidistant from a fixed center. The unit is organized so that students experience this. They establish the definition on day one, and from then on, every theorem we prove is a logical consequence of that definition combined with tools they already have. By the end, students should be able to point to almost any result in the unit and trace the chain of reasoning back to the same root.

The unit moves in three arcs. The first Lessons build the foundation: the definition and equation of a circle, the universality of similarity for circles, and arc length and sector area as proportional reasoning rather than memorization. The second arc is the geometric spine of the unit: the inscribed angle theorem and its corollaries, then the chord-bisector and radius-tangent theorems. These two lessons are unified by a recurring story problem, the archaeologist's broken pottery shard, which threads across both days and turns the abstract theorems into tools for solving a concrete and intuitively appealing question. The third arc closes a loop students did not know was open. The Law of Sines, which they derived and used in Unit 7, turns out to be a theorem about the circle circumscribed around the triangle: that mysterious common ratio a/sin A equals the diameter of that circle. The unit closes with a self-directed review day and an assessment.

The connections across units are denser here than in any other chapter of the book. Lesson 8.1 uses the distance formula from Unit 1 and the Pythagorean theorem from before that. Lesson 8.2 explicitly introduces the transformations frame for similarity, while honoring the Euclidean definition students built in Unit 6. Lesson 8.3 cashes in Lesson 8.2 immediately. Lesson 8.4's inscribed angle proof rests on the isosceles triangle and exterior angle theorems from Unit 3. Lesson 8.5's chord-bisector theorem is the Unit 4 perpendicular bisector theorem applied to two points on a circle — students who remember Unit 4 should find the proof almost immediate. Lesson 8.6's main result is a meeting point of Unit 6 (similarity, which produces the Law of Sines), Unit 7 (the Law of Sines itself), and the whole of Unit 8 (the circle theorems that interpret it). The book's recurring claim — that geometry is a connected web of consequences, not a list of disconnected facts — is most visibly true in this unit.

On Introducing Transformations Here

Unit 6 established similarity through the classical Euclidean lens — corresponding angles equal, corresponding sides in a constant ratio. That definition is precise, useful, and exactly the right tool for polygons. It is the wrong tool for circles. What are the corresponding sides of two circles? What are the corresponding angles? The Unit 6 language does not apply cleanly to a curve.

Lesson 8.2 introduces the transformations frame for similarity as a second, equivalent definition: two figures are similar if one can be mapped onto the other by a sequence of rigid motions and a dilation. The two definitions describe the same relationship. The transformations frame is the natural one for circles, and it will be the natural frame for several theorems that follow. Students do not formally prove the equivalence of the two definitions in this course. What they need here is to see that both frames exist, to use the transformations one when it helps, and to understand that it is not a replacement for what they own from Unit 6 but another lense.

Introducing transformations inside a circles unit, rather than in their own unit, is a deliberate choice. The argument for it is that the transformations frame earns its keep in the moment it is introduced: students immediately use it to prove that any two circles are similar, and they immediately apply that result to derive the arc length and sector area formulas the next day. The frame is not abstract scaffolding waiting for an application, it solves the problem at hand. The cost is that the transformations are introduced informally, without the formal apparatus of translation vectors, rotation matrices, or reflection axes. That formal work is deferred to its own unit. The bet is that students who use transformations meaningfully here will arrive at the formal unit with a foundation that makes the formalism feel like notation for something they already understand.

Lesson Sequence Overview

Lesson 8.1: What Is a Circle? — string task, the definition as set of equidistant points, derivation of the equation $(x-h)^2 + (y-k)^2 = r^2$ from the distance formula, extension ladder up to the three-points problem (1 day)

Lesson 8.2: All Circles Are Similar — introducing the transformations frame, two-task structure (three triangles first, then circles), the two-step similarity argument, the one-parameter idea (1 day)

Lesson 8.3: Arc Length and Sector Area — the fraction-times-whole framing, formulas derived rather than memorized, forward and reverse problems, the universality of π as a consequence of similarity (1 day)

Lesson 8.4: Central and Inscribed Angles — the broken shard launch, the inscribed angle theorem discovered inductively and proved deductively, Thales' theorem and the inscribed quadrilateral theorem as corollaries, the paper-corner trick (1 day)

Optional Enrichment Lesson: Angles Inside and Outside the Circle — a one-day extension between 8.4 and 8.5 that generalizes the inscribed angle theorem into three cases (vertex on, inside, or outside the circle). Not load-bearing for anything that follows; included for classes that have time and momentum

Lesson 8.5: Tangent Lines and the Chord-Bisector Theorem — the small shard launch, the chord-bisector theorem as Unit 4 perpendicular bisector applied to a circle, the radius-tangent theorem via shortest distance, the unifying idea named explicitly (1 day)

Lesson 8.6: The Chord-Sine Connection — the open question from Unit 7, the chord-sine identity proved from inscribed angle theorem and Thales, the Law of Sines reinterpreted as a circle theorem, Area = ½ab·sin C from the same diagram (1 day)

Lesson 8.7: Review Day — the satellite problem as a shared opening task, structured self-assessment on the rubric, self-directed practice from a problem bank organized by row and level (1 day)

Lesson 8.8: Assessment