

## Check Your Understanding 4.1

| Basic                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Intermediate                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Advanced                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| Triangle and Polygon Angle Relationships<br>→ Proving the laws                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <p><i>Basic</i></p> <p>1. Find each missing angle.</p> <p>(a) A triangle has angles <math>47^\circ</math> and <math>83^\circ</math>. Find the third angle.</p> <p>(b) A triangle has angles <math>2x^\circ</math>, <math>(x + 15)^\circ</math>, and <math>45^\circ</math>. Find <math>x</math>.</p> <p>(c) State the triangle angle sum theorem as a conditional statement.</p> <p>2. Find the interior angle sum of:</p> <p>(a) A pentagon (b) An octagon (c) A 12-sided polygon</p> | <p><i>Intermediate</i></p> <p>3. An exterior angle of a triangle measures <math>112^\circ</math>. One non-adjacent interior angle is <math>48^\circ</math>. Find the other non-adjacent interior angle.</p> <p>4. In triangle PQR, side QR is extended to point S. <math>\angle PQR = (3x + 10)^\circ</math>, <math>\angle QPR = (x + 20)^\circ</math>, and <math>\angle PRS = (5x - 6)^\circ</math>. Find <math>x</math> and all three interior angles of <math>\triangle PQR</math>.</p> <p>5. A student says: 'I tore the corners off a triangle and placed them together — they made a straight line.' Is this a deductive proof? What type of reasoning is it? What assumption does the formal proof silently require?</p> | <p><i>Advanced</i></p> <p>6. A student claims: 'I can find the interior angle sum of any polygon by placing a point inside and connecting to every vertex, then subtracting <math>360^\circ</math>.'</p> <p>(a) Is this method valid? Why does it work?</p> <p>(b) Show algebraically that this formula gives the same result as the triangulate-from-one-vertex method.</p> <p>(c) Each interior angle of a regular <math>n</math>-gon is <math>150^\circ</math>. Find <math>n</math>.</p> <p>7. Use the triangle angle sum theorem to prove the exterior angle theorem. Write a complete paragraph or flowchart proof.</p> |
| <p><b>Algebra Spiral</b><br/>Expand</p>                                                                                                                                                                                                                                                                                                                                                                                                                                               | <p><i>Algebra Spiral</i></p> <p>Expand each expression completely.</p> <p>(a) <math>(x - 5)(x + 8)</math></p> <p>(b) <math>(3a - 2)^2</math></p> <p>(c) <math>(2y + 1)(4y - 3)</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

**Answers — CYU 4.1****Triangle and Polygon Angles**

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**1a.**  $50^\circ$

**1b.**  $x = 40$ . Angles:  $80^\circ$ ,  $55^\circ$ ,  $45^\circ$ .

**1c.** If a polygon is a triangle, then the sum of its interior angles is  $180^\circ$ .

**2a.**  $540^\circ$    **2b.**  $1080^\circ$    **2c.**  $1800^\circ$

**3.**  $64^\circ$

**4.**  $x = 36$ .  $\angle PQR = 118^\circ$ ,  $\angle QPR = 56^\circ$ ,  $\angle PRQ = 6^\circ$ .

**5.** Inductive reasoning — empirical observation, not deductive proof. Formal proof assumes a parallel line can be constructed through the apex vertex.

**6a.** Valid.  $n$  triangles, total  $180n^\circ$ , subtract  $360^\circ$  at center  $\rightarrow 180n - 360$ .

**6b.**  $180(n - 2) = 180n - 360$ . ✓

**6c.**  $n = 12$ .

**7.**  $\angle ACD + \angle ACB = 180^\circ$  (linear pair).  $\angle A + \angle B + \angle ACB = 180^\circ$  (triangle angle sum). Both equal  $180^\circ$ , so  $\angle ACD + \angle ACB = \angle A + \angle B + \angle ACB$ . Subtract  $\angle ACB$ :  $\angle ACD = \angle A + \angle B$ . ■

**Algebra Spiral**

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**(a)**  $x^2 + 3x - 40$

**(b)**  $9a^2 - 12a + 4$

**(c)**  $8y^2 - 2y - 3$