# Check Your Understanding 4.4

*Show all work. Write CPCTC in full before using it. For proofs: flowchart or paragraph form only.*

| Basic | Intermediate | Advanced |
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| **Row 2 Spiral — Constructions**<br>*→ Building the tools* | | |
| *Basic — Spiral*<br>1. Describe how to construct the perpendicular bisector of segment AB using only compass and straightedge. Which constructions from earlier lessons does this use? | *Intermediate — Spiral*<br>2. Construct the perpendicular bisector of the segment below. Show all arc marks and label the midpoint M and the two arc intersections.<br>Identify the triangle congruence that proves the construction works. | *Advanced — Spiral*<br>3. In Lesson 4.5 practice Problem 6 in the construction row, you proved the altitude from the apex of an isosceles triangle is also the perpendicular bisector of the base. Explain which specific step of the altitude proof gives you the midpoint. Then state the full claim as a conditional. |
| **Row 3 Spiral — Base Angles Theorem**<br>*→ Proving and earning the converse* | | |
| *Basic — Spiral*<br>4. In isosceles triangle ABC with AB = AC, the angle bisector from A meets BC at D. Without using the perpendicular bisector theorem, explain why AD must be perpendicular to BC. | *Intermediate — Spiral*<br>*5. In isosceles triangle PQR with PQ = PR, ∠QPR = (6x − 8)° and ∠PQR = (2x + 16)°. Find x and all three angles.* | *Advanced — Spiral*<br>**6. Given: △ABC with AB = BC. M is the midpoint of AC. Prove that BM ⊥ AC.** |

| Basic | Intermediate | Advanced |
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## Bisector Theorems

*→ Turning constructions into proofs*

*Basic*

*7. Point P lies on the bisector of ∠ABC. The perpendicular distance from P to side BA is 9. What is the perpendicular distance from P to side BC?*

*8. Point Q lies on the perpendicular bisector of segment XY. If QX = 13, what is QY?*

9. State the angle bisector theorem as a conditional statement.

10. State the converse of the perpendicular bisector theorem.

*Intermediate*

*11. In ∠DEF, the angle bisector passes through point P. The perpendicular distance from P to DE is $(3x + 2)$ and to EF is $(5x - 8)$. Find x and the perpendicular distance.*

*12. Point P is equidistant from A(0, 0) and B(8, 0). Find the equation of the set of all such points. What geometric object is this?*

*13. In triangle ABC, a point D on BC has perpendicular distance 5 to AB and lies 7 units from BC. Can D lie on the angle bisector from A? Explain.*

*Advanced*

14. Prove the perpendicular bisector theorem. State what is given, what is to be proven, and cite every reason.

15. A point P is equidistant from the sides of ∠ABC and also equidistant from B and C. What can you conclude about P? Justify using both bisector theorems.

**16. Given: Point P is equidistant from A and B, and also equidistant from B and C. Prove that P is equidistant from A and C.**

## Algebra Spiral

*Solve systems of two equations*

*Algebra Spiral*

Solve each system. Show your method clearly.

(a) $2x + y = 10$ and $x - y = 2$

(b) $y = 3x - 4$ and $2x + 3y = 13$

## Answers — CYU 4.4

### Row 2 Spiral

**1.** Set compass wider than half AB. Draw arcs from A above and below AB. Same width from B, crossing first arcs. Connect intersections. Uses the same compass arc technique as the altitude construction (Lesson 4.2).

**2.** Construction check — arcs clearly drawn, midpoint M labeled. SAS: AM = MB (midpoint), ∠PMA = ∠PMB = 90° (perpendicular), PM = PM (reflexive). △PMA ≅ △PMB.

**3.** In the altitude proof, △APH ≅ △AQH by SAS → PH = QH by CPCTC. This makes H the midpoint of PQ. Since the altitude is also perpendicular, it is the perpendicular bisector. Conditional: If a triangle is isosceles, then the altitude from the apex is also the perpendicular bisector of the base.

### Row 3 Spiral

**4.** Draw angle bisector AD. AB = AC (given), ∠BAD = ∠CAD (bisector), AD = AD (reflexive). △ABD ≅ △ACD (SAS). ∠ADB = ∠ADC (CPCTC). Linear pair → each = 90°. AD ⊥ BC.

**5.** PQ = PR → ∠Q = ∠R. So 2x + 16 = base angle. Apex + 2(base) = 180°: (6x−8) + 2(2x+16) = 180 → 6x−8+4x+32=180 → 10x=156 → x=15.6. ∠QPR = 85.6°. ∠Q = ∠R = 47.2°.

**6.** AB = BC → ∠BAC = ∠BCA (base angles). AM = MC (midpoint). △BAM and △BCM: AB = BC (given), AM = MC, ∠BAM = ∠BCM (base angles). △BAM ≅ △BCM (SAS). ∠BMA = ∠BMC (CPCTC). Linear pair → each = 90°. BM ⊥ AC. ■

### Bisector Theorems

**7.** 9.

**8.** 13.

**9.** If a point lies on the bisector of an angle, then its perpendicular distance to each side of the angle is equal.

**10.** If a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of that segment.

**11.** 3x + 2 = 5x − 8 → x = 5. Perpendicular distance = 17.

**12.** x = 4. Vertical line — the perpendicular bisector of AB.

**13.** No. Angle bisector requires equal perpendicular distances to both sides. 5 ≠ 7.

**14.** Given: l ⊥ AB at midpoint M, P on l. AM = MB (midpoint). ∠PMA = ∠PMB = 90° (l ⊥ AB). PM = PM (reflexive). △PMA ≅ △PMB (SAS). PA = PB (CPCTC). ■

**15.** Equidistant from sides → P on angle bisector from B (converse angle bisector theorem). Equidistant from B and C → P on perpendicular bisector of BC (converse perpendicular bisector theorem). P lies on both bisectors.

**16.** PA = PB (given). PB = PC (given). PA = PC by Common Notion 1. ■

### Algebra Spiral

**(a)** x = 4, y = 2.

**(b)** x = 25/11, y = 41/11.