

What Makes an Argument Good?

Unit 2: Logic and Reasoning • Day 6 of 11 • Fallacies, Venn Diagrams, Connection to Conditional Forms

At a Glance

Pacing: Monty Python activator (20 min) • Guided notes + Venn diagrams (25 min) • Return to argument table (10 min) • CYU (35 min)

Materials: Boards and markers • YouTube clip (see below) • Extended argument table (add Structure and Type columns)

Opening: Distribute student notes sheet before guided notes. Students attempt last two columns at boards first, then fill in after class review.

Activator — The Witch Trial (≈20 min)

Video

Monty Python and the Holy Grail (1975) — ‘She’s a Witch!’ scene. Search YouTube: ‘Monty Python Holy Grail She’s a Witch’. Approximately 3 minutes.

YouTube link: <https://youtu.be/rf71YotfykQ?si=DuxIJ5DzchRpNx1T>

After watching, put the argument chain on the board. Ask: ‘Is this a valid argument? Is it sound? Where does it go wrong?’

The Witch Trial — Four Arguments to Evaluate

Argument 1 — P1: All witches can be burned. P2: All things made of wood can be burned. ∴ All witches are made of wood.

Argument 2 — P1: All things made of wood float. P2: All things that weigh the same as a duck float. ∴ Things that weigh the same as a duck are made of wood.

Argument 3 — P1: All witches are made of wood. P2: All things that weigh the same as a duck are made of wood. ∴ Witches weigh the same as a duck.

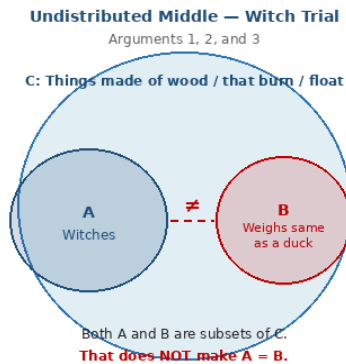
Argument 4 — P1: All witches weigh the same as a duck. P2: This thing weighs the same as a duck. ∴ This thing is a witch.

Draw Venn diagrams for each fallacy as you discuss them (next page)

Venn Diagrams — Why Each Argument Fails

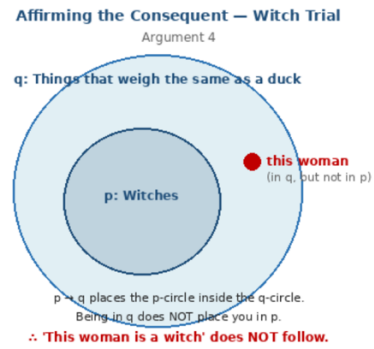
Draw these on the board as you discuss each argument. The diagrams show why sharing a category does not establish equivalence, and why affirming the consequent fails.

Arguments 1, 2, 3 — Undistributed Middle



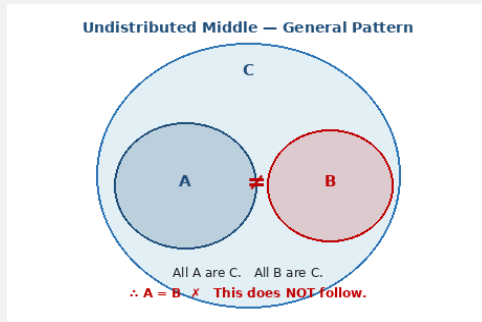
Both A and B are subsets of C. That does not mean $A = B$. The two inner circles can sit anywhere inside the outer one — they may not overlap at all.

Argument 4 — Affirming the Consequent



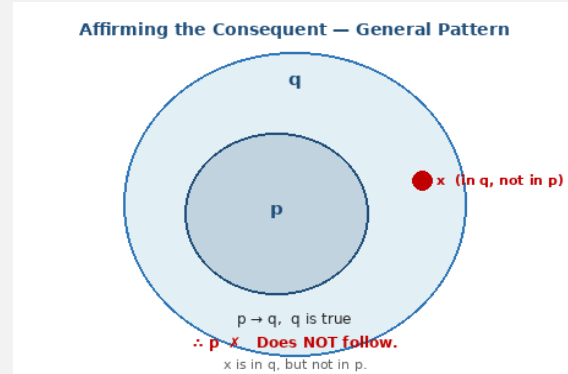
If $p \rightarrow q$, then the p-circle sits inside the q-circle. This thing is in the q-circle (weighs same as duck) — but that does not put it in the p-circle (witch). There is space in the outer ring not covered by the inner one.

The General Pattern — Undistributed Middle



All A are C. All B are C. ∴ $A = B$ — This does not follow. Sharing a common superset tells us nothing about the relationship between A and B.

The General Pattern — Affirming the Consequent



$p \rightarrow q$, q is true, ∴ p — This does not follow. The dot is inside q, but not inside p. Being in q does not guarantee being in p.

GUIDED NOTES — ARGUMENT FORMS AND FALLACIES

State directly before beginning: 'The names we are about to learn are not new ideas. They are names for argument patterns you already understand — specifically, what happens when you use the wrong form of a conditional statement from Lesson 2.2b.'

(the student note sheet matches this with the last two columns to be filled in)

Name	Symbolic Form	Example	Connection to 4 Forms
Law of Detachment ✓	$p \rightarrow q$ p $\therefore q$	<i>If it rains, ground gets wet. It is raining. $\therefore$ Ground is wet.</i>	Uses the conditional $p \rightarrow q$ directly.
Denying the Consequent ✓	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	<i>If it rains, ground gets wet. Ground is not wet. $\therefore$ It is not raining.</i>	Uses the contrapositive $\sim q \rightarrow \sim p$. Valid because the contrapositive is equivalent to the original.
Affirming the Consequent ✗ (Fallacy)	$p \rightarrow q$ q $\therefore p$ ✗	<i>If I eat broccoli, I'll be healthy. I am healthy. $\therefore$ I must have eaten broccoli.</i>	Treats the converse ($q \rightarrow p$) as valid. The converse is not equivalent to the original.
Denying the Antecedent ✗ (Fallacy — Inverse Error)	$p \rightarrow q$ $\sim p$ $\therefore \sim q$ ✗	<i>If it doesn't rain, we'll go outside. It rained. $\therefore$ We won't go outside.</i>	Treats the inverse ($\sim p \rightarrow \sim q$) as valid. Named the 'inverse error' in 2.2b — now has its formal name.
Undistributed Middle ✗ (Fallacy)	All A are C. All B are C. $\therefore A = B$ ✗	<i>All dogs are animals. All cats are animals. $\therefore$ All cats are dogs.</i>	Sharing a trait doesn't establish equivalence. Venn diagram: $A \subset C$ and $B \subset C$ does not mean $A = B$. <i>Not one of the four forms — a distinct structural error.</i>

The key insight: fallacies are the wrong conditional forms

Conditional ($p \rightarrow q$): Law of Detachment — correct.

Contrapositive ($\neg q \rightarrow \neg p$): Denying the Consequent — correct.

Converse ($q \rightarrow p$): Affirming the Consequent — fallacy. The converse is not equivalent.

Inverse ($\neg p \rightarrow \neg q$): Denying the Antecedent — fallacy. The inverse is not equivalent.

Students who see this do not need to memorize four separate patterns. They need to remember one thing from 2.2b: only the conditional and its contrapositive are logically equivalent.

 Students make notes on:

- Five argument forms table: name, symbolic form, example, connection to four conditional forms (student sheet provided)
- Valid/sound/invalid definitions
- Venn diagrams for affirming the consequent and undistributed middle

Return to the Argument Table

Distribute the extended argument table. Students complete the Structure and Type columns using their new vocabulary. This is the same nine arguments — not a new task. Groups should move quickly. Emphasize F and H when reviewing: both are valid but not sound, which students originally may have called invalid.

ID	Premises	Conclusion	Structure	Valid/Sound/Invalid
A	P1: All cats are mammals. P2: Felix is a cat.	Felix is a mammal.		
B	P1: All dogs are animals. P2: All cats are animals.	Therefore, all cats are dogs.		
C	P1: If it rains, the ground gets wet. P2: It is raining.	The ground is wet.		
D	P1: If I eat broccoli, I'll be healthy. P2: I am healthy.	I must have eaten broccoli.		
E	P1: All humans are mortal. P2: Socrates is mortal.	Therefore, Socrates is a human.		
F	P1: All unicorns have horns. P2: Steve is a unicorn.	Steve has a horn.		
G	P1: If aliens exist, then Area 51 is hiding them. P2: Aliens exist.	Therefore, Area 51 is hiding them.		
H	P1: If the moon is made of cheese, then it tastes delicious. P2: The moon is made of cheese.	Therefore, it tastes delicious.		
I	P1: All Martians wear hats. P2: Fred is a Martian.	Fred wears a hat.		