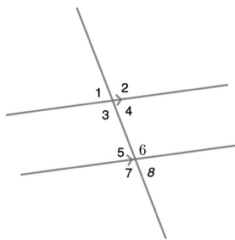
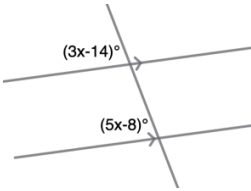
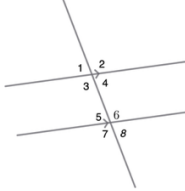


## Check Your Understanding 2.5

Show all work. Proofs must be written in flowchart or paragraph form — no two-column proofs.

| Basic                                                                                                                                                                                                                                                                                               | Intermediate                                                                                                                                                                                                                                                                                                                                                      | Advanced                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| <b>Row 2 Spiral — Deductive Reasoning and Conditional Statements</b><br>→ From experience to necessity                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>Basic — Spiral</i><br>1. Write each statement in conditional (if-then) form. Label the hypothesis $p$ and the conclusion $q$ .<br><br>(a) <i>All right angles are congruent.</i><br>(b) <i>Parallel lines never intersect.</i><br>(c) <i>Supplementary angles sum to <math>180^\circ</math>.</i> | <i>Intermediate — Spiral</i><br>2. Identify the hypothesis and conclusion of each conditional. Then state whether the conditional is true or false. If false, give a counterexample.<br><br>(a) <i>If two angles are supplementary, then they form a linear pair.</i><br>(b) <i>If a triangle has a right angle, then the other two angles are complementary.</i> | <i>Advanced — Spiral</i><br>3. In Lesson 2.2a you learned that deductive reasoning forces a conclusion to be true — not just probably true. Write a deductive argument with three steps showing that if two lines are cut by a transversal and alternate interior angles are equal, then co-interior angles must be supplementary. Use the law of syllogism and cite the angle relationship you are using at each step.            |
| <b>Row 3 Spiral — The Four Conditional Forms</b><br>→ Earning the contrapositive                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>Basic — Spiral</i><br>4. Write all four forms of the conditional. State whether each is true or false.<br><br><b><i>'If two angles are vertical angles, then they are congruent.'</i></b>                                                                                                        | <i>Intermediate — Spiral</i><br>5. A student writes: 'The contrapositive of a conditional is always true whenever the conditional is true.' Is this correct? Explain why using one of the four forms from this unit.<br>6. Write a true conditional from geometry whose converse is false. Explain why the converse fails.                                        | <i>Advanced — Spiral</i><br>7. You learned in Lesson 2.2b that $p \rightarrow q$ and $\sim q \rightarrow \sim p$ are logically equivalent. Use this equivalence to explain why the following is a valid argument. Name the argument form.<br><i>If two lines are parallel, then alternate interior angles are equal.</i><br><i>Alternate interior angles are NOT equal.</i><br>$\therefore$ <i>The two lines are not parallel.</i> |
| <b>Row 4 Spiral — Valid Arguments and Fallacies</b><br>→ Structure over content                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>Basic — Spiral</i><br>8. Identify each argument as valid or a fallacy. Name the form.<br><br>(a) <i>If it is a square, then it has four right angles. <math>ABCD</math> has four right angles. <math>\therefore ABCD</math> is a square.</i>                                                     | <i>Intermediate — Spiral</i><br>9. A student makes this argument: 'Corresponding angles are equal when lines are parallel. These angles are equal. Therefore the lines must be parallel.' Identify the argument form and determine whether it is valid.                                                                                                           | <i>Advanced — Spiral</i><br>11. In the Unit 2.4 Lewis Carroll puzzle, the law of syllogism was used multiple times in sequence. Explain why the law of syllogism is the engine of deductive reasoning in geometry — why does every proof chain depend on it, even when the steps are not written in $p \rightarrow q$ form?                                                                                                        |

| Basic                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Intermediate                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Advanced                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(b) If a polygon is regular, then all its sides are equal. Polygon EFGH is not regular. <math>\therefore</math> Its sides are not all equal.</p> <p>(c) If <math>\angle A = \angle B</math> and <math>\angle B = \angle C</math>, then <math>\angle A = \angle C</math>.</p>                                                                                                                                                                                                                                                                                                                                                                        | <p>10. Use the law of syllogism to chain these statements into one conclusion.</p> <p><math>p \rightarrow q</math>: If two lines are cut by a transversal with equal corresponding angles, then the lines are parallel.</p> <p><math>q \rightarrow r</math>: If two lines are parallel, then co-interior angles are supplementary.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| Transversal Angle Relationships<br>→ The first deductive geometry                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <p><i>Basic</i></p> <p>12. In the diagram, lines <math>m</math> and <math>n</math> are cut by transversal <math>t</math>. Name one pair of each type of angle relationship.</p>  <p>(a) Corresponding angles<br/>(b) Alternate interior angles<br/>(c) Alternate exterior angles<br/>(d) Co-interior (same-side interior) angles</p> <p>13. Lines <math>m</math> and <math>n</math> are parallel. <math>m\angle 2 = 118^\circ</math>. Find <math>m\angle 6</math>, <math>m\angle 3</math>, and <math>m\angle 5</math>. State the relationship you used for each.</p> | <p><i>Intermediate</i></p> <p>14.</p>  <p>(a) What relationship do <math>\angle 1</math> and <math>\angle 5</math> have if <math>p \parallel q</math>? Write the conditional.</p> <p>(b) Assuming <math>p \parallel q</math>, find <math>x</math> and both angle measures.</p> <p>15. Lines <math>a</math> and <math>b</math> are parallel. <math>\angle 3 = (7x - 10)^\circ</math> and <math>\angle 6 = (3x + 30)^\circ</math>.</p> <p>(a) What relationship do <math>\angle 3</math> and <math>\angle 6</math> have? Write the conditional.</p> <p>(b) Find <math>x</math> and both angles.</p> <p>16. <math>\angle 4 = (2x + 15)^\circ</math> and <math>\angle 5 = (4x - 9)^\circ</math> are co-interior angles, and the lines are parallel. Find <math>x</math> and show why this must be true.</p> | <p><i>Advanced</i></p> <p>17. <b>Given:</b> lines <math>m</math> and <math>n</math> are cut by transversal <math>t</math>. <math>\angle 1</math> and <math>\angle 5</math> are corresponding angles and <math>\angle 1 \cong \angle 5</math>.</p>  <p><b>Prove that ALL other pairs of corresponding angles are also congruent.</b></p> <p>Your proof must: use only results already established in this course (vertical angles theorem, linear pair theorem, or the given corresponding angle congruence); cite every reason; be written in flowchart or paragraph form.</p> |
| <p><b>Algebra Spiral</b><br/>Exponent Properties — Quotient Rule and Negative Exponents</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | <p><i>Algebra Spiral</i></p> <p>Simplify each expression. Write your answer without negative exponents.</p> <p>(a) <math>x^8 \div x^3</math><br/>(b) <math>(2x^3y^2) / (4x^5y)</math><br/>(c) <math>3x^{-2} \cdot x^4</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

**Answers — CYU 2.5****Row 2 Spiral — Deductive Reasoning and Conditional Statements**

- 1a.** If a figure is a right angle, then it is congruent to every other right angle.
- 1b.** If two lines are parallel, then they never intersect.
- 1c.** If two angles are supplementary, then they sum to  $180^\circ$ .
- 2a.** p: two angles are supplementary. q: they form a linear pair. FALSE — two supplementary angles can be non-adjacent (e.g.,  $120^\circ$  and  $60^\circ$  in different parts of a diagram).
- 2b.** p: a triangle has a right angle. q: the other two angles are complementary. TRUE — remaining angles sum to  $90^\circ$ .
- 3.** Step 1: If alt. int. angles are equal, then the lines are parallel (contrapositive of the given). Step 2: If lines are parallel, then co-interior angles are supplementary (transversal theorem). Step 3: Law of Syllogism: if alt. int. angles are equal, then co-interior angles are supplementary. ■

**Row 3 Spiral — The Four Conditional Forms**

- 4.** Conditional: if vertical angles, then congruent. TRUE. Converse: if congruent, then vertical. FALSE (e.g., two separate  $90^\circ$  angles). Inverse: if not vertical, then not congruent. FALSE. Contrapositive: if not congruent, then not vertical. TRUE.
- 5.** Correct. The conditional and its contrapositive are logically equivalent — they always have the same truth value. This is why denying the consequent is a valid argument form.
- 6.** Accept any valid example, e.g. 'If a triangle is equilateral, then it is isosceles.' Converse: 'If isosceles, then equilateral' — false for any non-equilateral isosceles triangle.
- 7.**  $p \rightarrow q$  and  $\sim q \rightarrow \sim p$  are equivalent. Here  $p$  = parallel,  $q$  = alt. int. angles equal. Given  $\sim q$  (not equal)  $\rightarrow \sim p$  (not parallel). This is Denying the Consequent (Modus Tollens). ✓

**Row 4 Spiral — Valid Arguments and Fallacies**

- 8a.** Affirming the Consequent ✗ — fallacy.
- 8b.** Denying the Antecedent ✗ — fallacy. Not regular does not mean sides are unequal (irregular polygons can still have equal sides).
- 8c.** Law of Syllogism ✓ — this is the transitivity of equality.
- 9.** Affirming the Consequent ✗. The student confirmed  $q$  (equal angles) and concluded  $p$  (parallel lines). This is the converse, which is not guaranteed by the original conditional alone.
- 10.** By the Law of Syllogism: If two lines are cut by a transversal with equal corresponding angles, then co-interior angles are supplementary.
- 11.** Every step in a geometric proof connects one established fact to another using a conditional. The law of syllogism is exactly this: if  $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow C$ . Each line of a proof is one application of the law of syllogism. A proof is nothing more than a chain of syllogisms from the given to the conclusion.

**Transversal Angle Relationships**

- 12.** Accept any correct pairs from the diagram. Corresponding: e.g.  $\angle 1$  and  $\angle 5$ . Alt. interior: e.g.  $\angle 3$  and  $\angle 6$ . Alt. exterior: e.g.  $\angle 1$  and  $\angle 8$ . Co-interior: e.g.  $\angle 3$  and  $\angle 5$ .

**13.**  $\angle 6 = 118^\circ$  (corresponding angles,  $m \parallel n$ ).  $\angle 3 = 62^\circ$  (linear pair with  $\angle 2$ ).  $\angle 5 = 62^\circ$  (co-interior angles sum to  $180^\circ$ , or alternate interior angles with  $\angle 3$ ).

**14a.** Corresponding angles. If  $p \parallel q$ , then corresponding angles are congruent:  $\angle 1 = \angle 5$ .

**14b.**  $3x + 14 = 5x - 8 \rightarrow x = 11$ . Both angles =  $47^\circ$ .

**15a.** Co-interior (same-side interior) angles. If  $a \parallel b$ , then co-interior angles are supplementary:  $\angle 3 + \angle 6 = 180^\circ$ .

**15b.**  $(7x - 10) + (3x + 30) = 180 \rightarrow 10x + 20 = 180 \rightarrow x = 16$ .  $\angle 3 = 102^\circ$ ,  $\angle 6 = 78^\circ$ .

**16.** Co-interior angles are supplementary:  $(2x+15)+(4x-9) = 180 \rightarrow 6x + 6 = 180 \rightarrow x = 29$ .  $\angle 4 = 73^\circ$ ,  $\angle 5 = 107^\circ$ . They sum to  $180^\circ$ . ✓

**17.** Given:  $\angle 1 \cong \angle 5$  (corresponding angles, given).  $\angle 1 \cong \angle 3$  (vertical angles theorem).  $\angle 5 \cong \angle 7$  (vertical angles theorem).  $\angle 2$  is supplementary to  $\angle 1$  (linear pair).  $\angle 6$  is supplementary to  $\angle 5$  (linear pair). Since  $\angle 1 \cong \angle 5$ , and  $\angle 2$  and  $\angle 6$  are each supplementary to congruent angles,  $\angle 2 \cong \angle 6$ . Similarly  $\angle 3 \cong \angle 1 \cong \angle 5 \cong \angle 7$ , so  $\angle 3 \cong \angle 7$ . And  $\angle 4 \cong \angle 2 \cong \angle 6 \cong \angle 8$ . All four pairs of corresponding angles (1,5), (2,6), (3,7), (4,8) are congruent. ■

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### Algebra Spiral

(a)  $x^5$

(b)  $y / (2x^2)$

(c)  $3x^2$