

Specializing and Testing

Unit 0: Problem Solving and Mathematical Habits • Day 1 of 4

At a Glance

Pacing: ~45 min total for class today (1st day)

Materials: Boards and markers • Visibly random group assignments ready before class

Mathematical habits introduced today: Specializing — trying specific examples • Testing — looking for counterexamples

BTC setup note: This is Day 1. Establish visibly random groups and vertical non-permanent surfaces before students arrive. No seating plan. Students stand at boards from the first task.

OBJECTIVES

By the end of this lesson students:

- Have experienced collaborative problem solving at vertical boards.
- Can explain what specializing means and why it is a useful starting point.
- Understand that testing specific examples does not prove a general rule — and that one counterexample disproves it.
- Have encountered confirmation bias as a named idea.

TASKS

Task 1 — Warehouse Discount (≈25 min)

Say the prompt verbally only — do not write it on the board.

Prompt

In a warehouse you get a 20% discount but must pay 15% sales tax. Which order would you prefer them calculated in?

Key teacher moves: circulate and note strategies. Do not hint or confirm. Once groups have a result, push:

- Does your result hold for other starting prices? Other discount and tax rates?
- Can you prove it works for any price?

Resolution

$P(0.8)(1.15) = 0.92P$ regardless of order. Multiplication is commutative and associative — you are multiplying three numbers, order never matters. The algebraic generalization is the proof.

Extension ladder: Generalize for any discount d and tax t . What if discount and tax are equal?

Task 2 — Guess My Rule (≈20 min)

Setup

Say: 'I have a rule. I will give you a 3-number sequence that fits my rule. You may test your ideas by giving me your own 3-number sequence and I will tell you if it fits. You may not ask me what the rule is.'

Starting sequence: 6 – 8 – 10

Actual rule: numbers in increasing order.

⚠ **The teaching point is the pattern of inquiry, not the rule**

Most students only test sequences that confirm what they already think the rule is. They will not try to disprove their idea. Let this pattern run for a while before asking: 'Would you ever learn you were wrong if you kept asking things that matched your guess?'

After groups discover the rule or are ready to discuss, ask:

- What did you think the rule was?
- Did you try anything that could disprove your idea?
- How many confirming examples would it take to be certain?
- What single sequence would have disproved your first guess?

CONSOLIDATION

The goal is the thinking process, not the answers. Ask how groups started, whether anyone felt sure before testing, and what made them confident in their final answer.

For the warehouse problem: walk through the algebraic generalization. This is the first time a variable is used to show something is true 'for every' rather than for a specific case. Name that move explicitly.

Two key terms to introduce and post — these recur all year

Confirmation bias: the tendency to look only for evidence that supports our existing belief.

Counterexample: one specific case that disproves a general statement. A million confirming examples do not prove a claim. One counterexample disproves it.

Post these visibly in the room. They will be named again in Unit 2 and return every time students make conjectures.

 Students make notes on:

- Confirmation bias: definition
- Counterexample: definition and what it means for general claims
- Specializing: what it means and why it is a useful starting point

Feeds into

- Lesson 0.2: organizing and generalizing build directly on today's specializing
- Unit 2, Lesson 2.1: the circle task revisits counterexample and confirmation bias formally
- Every unit: 'failing to find a counterexample does not prove a claim' is restated throughout the course