

Axioms and Shared Premises

Unit 2: Logic and Reasoning • Day 9 of 11 • Sudoku Task • Why Careful Thinkers Disagree • Euclid's 10 Axioms

At a Glance

Pacing: Sudoku task + gallery walk (40 min) • Discussion: why careful thinkers disagree (10 min) • Aristotle/Franklin/Jefferson/Euclid brief (5 min) • Euclid's 10 axioms guided notes (30 min)

Materials: Sudoku puzzle printed • Boards and markers • Student notes sheet (axioms table)

Key instruction: Do not reveal the connection between the Sudoku and the axioms before the task. Let it land during the discussion.

Nav instrument rows active: All rows spiral • Row 7 (Axioms and Shared Premises) activates today

OBJECTIVES

By the end of this lesson students can:

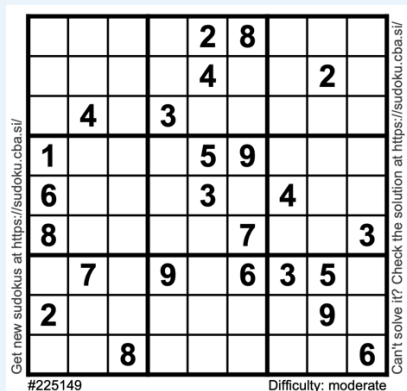
- Explain why shared premises are necessary for deductive reasoning to produce agreement.
- State Euclid's 5 postulates and 5 common notions and explain why each is accepted without proof.
- Connect the structure of an axiomatic system to the logic laws from earlier in the unit.

TASK — SUDOKU (≈35 MIN)

Sudoku Thinking Task

Task prompt

Solve as much of the Sudoku puzzle as you can. For every number you place, write a reason. Not just what you did — why you know it must go there. Your reasons should be specific enough that someone who cannot see your board could follow your logic.



Notation to introduce before the task begins — this significantly improves reasoning quality

Cell reference: row:column, numbered 1–9 from top-left. The center cell is 5:5.

Box reference: The nine 3×3 boxes numbered 1–9 left to right, top to bottom. Top-left is Box 1.

With this notation, a student can write: ‘Cell 3:7 must be 4 because row 3 already contains 1,2,3,5,6,7,8,9, and Box 3 already contains all digits except 4 — by disjunctive syllogism, 4 is the only remaining option.’

⚠ Watch for these logic laws — name them in the gallery walk

Disjunctive syllogism: ‘This cell’s row contains all digits except 4 and 7. Column 6 already has a 7. Therefore this cell must be 4.’ Exactly $p \vee q, \neg q \therefore p$.

Law of syllogism: ‘If this cell is 4, then cell 7:3 cannot be 4. If cell 7:3 cannot be 4, then it must be 6. Therefore if this cell is 4, then cell 7:3 is 6.’

Gallery walk: Look for groups who used shorthand or numbering rules as they model notation and axioms in geometry.

DISCUSSION — WHY CAREFUL THINKERS STILL DISAGREE

Two-step question — give each step room to breathe

Step 1: ‘Why do people still disagree about things?’ Accept the expected answer: because they don’t know or don’t apply logic correctly.

Step 2: ‘But here is a harder version: among people who are careful, who know the laws of logic, and who apply them correctly — why do some still reach very different conclusions about important things?’

When a student says some version of ‘they started from different premises’ — stop and mark it. Say: ‘Yes. That’s exactly it. Everything else in this lesson follows from what you just said.’ Then connect back to the Sudoku: everyone agreed because everyone started from the same three rules.

📺 Narrative Arc — Use after the central discussion question lands

“Everything else in this lesson follows from what you just said. Two careful, logically correct thinkers can reach different conclusions if they start from different premises. This is not a problem with logic. It is how deductive systems work.”

“Geometry solves this by naming the premises up front. Before the first proof, before the first theorem, we state exactly what we accept as true. That is what Euclid did. That is what we are about to do.”

Aristotle (~335 BC)	Franklin and Jefferson (1776)	Euclid (~300 BC)
Called it Demonstrative Science. True knowledge must be built by deductive reasoning from first principles — things so basic they need no proof.	Jefferson drafted the Declaration, but it was Franklin who changed 'sacred and undeniable' to 'self-evident.' The edit matters: self-evident truths require no external authority to justify them. They are accepted because they are accepted.	Built all of geometry from 10 starting points. His true genius was not discovering geometric facts — many were already known. It was showing that accepting just 10 self-evident premises makes all the others undeniable . Some estimates put the number of theorems derivable from those 10 premises at around 465.
All three are doing the same thing: naming what they accept before they start reasoning.		

EUCLID'S 10 AXIOMS — TEACHER REFERENCE

Walk through each axiom together. Distribute the student notes sheet. Students draw a diagram or write a note for each one.

The Five Postulates

	Statement	Teacher Note
P1	A straight line can be drawn connecting any two points.	<i>Immediately accepted. Ask: can you always draw exactly one such line?</i>
P2	A finite line segment can be extended indefinitely in either direction.	<i>Immediately accepted. Students will cite 'by Postulate 2' when extending segments in proofs.</i>
P3	A circle can be drawn with any given center point and any given radius.	<i>Immediately accepted.</i>
P4	All right angles are equal to one another.	<i>Euclid had no degree measures. He needed to assert that all right angles are the same. For us: 90° means 90° everywhere.</i>
P5	(Playfair's version) Through any point not on a given line, there is exactly one line parallel to the given line.	<i>The famous postulate. Euclid avoided using it for the first 28 propositions. Vote or discuss which version to adopt — Playfair's is usually easier to apply. Accept the co-interior angle addition as part of P5.</i>

The Five Common Notions

	Statement	Teacher Note
CN1	Things equal to the same thing are equal to each other. (If $a=c$ and $b=c$, then $a=b$.)	<i>Transitive property of equality. Immediately recognized.</i>
CN2	If equals are added to equals, the wholes are equal.	<i>Addition property of equality. Used constantly in proof.</i>
CN3	If equals are subtracted from equals, the remainders are equal.	<i>Subtraction property of equality.</i>

CN4	Things that coincide with one another are equal to one another.	Name the Reflexive Property here: any figure is congruent to itself. $AB \cong AB$ by the Reflexive Property.
CN5	The whole is greater than the part — and the whole equals the sum of its parts.	Expand to: Segment Addition (if B is between A and C, then $AB+BC=AC$) and Angle Addition. Accept these as part of CN5.

History worth noting: Postulate 5

Euclid himself seemed uneasy about P5 — he avoided using it for as long as possible. For 2000 years, mathematicians suspected it could be derived from the other four. In the 19th century, Bolyai, Lobachevsky, and Gauss independently showed it cannot: you can build entirely consistent geometries by replacing it. In hyperbolic geometry, infinitely many parallels exist through a point. In elliptic geometry (the sphere), none do. Both are used in physics and navigation.

This is exactly what the lesson's central question predicts: change the starting premises, reach different valid conclusions. The parallel postulate is the most famous example of this in all of mathematics.

Memorization note for students

Students are responsible for knowing all 10 axioms by number. When writing proofs they will cite them: 'by Postulate 2' or 'by Common Notion 1.' Study tip: don't re-read. Cover the list, say the axiom aloud, check yourself. Space practice over several days.

Students make notes on:

- Key idea: an axiom is accepted without proof — it is the starting point from which all other results are derived
- All 10 axioms with diagram/note space (student notes sheet)
- Memorization strategy noted

Narrative Arc — End of Lesson 2.5 — closing the unit arc

"We began this unit by watching inductive reasoning fail. We built the tools of deductive reasoning. We learned that even careful deductive thinkers can disagree — and we found out why. Two careful thinkers reasoning correctly from different starting premises will reach different conclusions. The resolution is to name the starting premises before we begin."

"Euclid named his: ten of them. Five postulates, five common notions. From those ten statements, accepted without proof, all 465 theorems in the Elements follow necessarily. The law of syllogism is the mechanism. The axioms are the fuel. Unit 3 is where we start burning it."