Lesson 3.3 — Teacher Reference Notes

Reading a Diagram

Unit 3: Proof and Congruence • Day 3 of 9

At a Glance	
	Pacing: Opening task + notes (~30 min) • Poster task (~55 min) • Closing consolidation (~5 min) **Materials:** Boards and markers • Mathmedic poster task cards printed (one set per group) • Large paper or poster board, markers, glue or tape • Teacher-prepared opening diagrams (see image placeholders) **Nav instrument rows active:** Rows 1–3 spiral • Row 4 (Reading a Diagram / CPCTC) activates today **Timing warning:** The opening task and notes must stay within 30 minutes. The poster task needs at least 55 minutes. Protect that time. **If groups do not finish the poster:** Allow 10 minutes at the start of Lesson 3.4 to complete it. Do not assign poster work as homework.

OBJECTIVES

By the end of this lesson students can:

- Identify what information is free from a diagram (vertical angles, linear pairs, shared sides) and what must be explicitly given.
- Explain why appearance alone cannot justify a claim in a proof.
- Write correct triangle congruence statements with proper correspondence notation.
- State CPCTC as a biconditional, recite the full definition before using the abbreviation, and list all six congruent statements that follow from a triangle congruence statement.
- Use free diagram information, congruence shortcuts, and CPCTC together to classify triangle pairs.

Narrative Arc — Before the opening task

“We can identify congruent triangles using shortcuts. But before we can apply any shortcut, we need to know what information we actually have. Some of it is written in the given. Some of it we can read directly from the diagram — without being told. And some of it we want to assume because it looks true but cannot.”

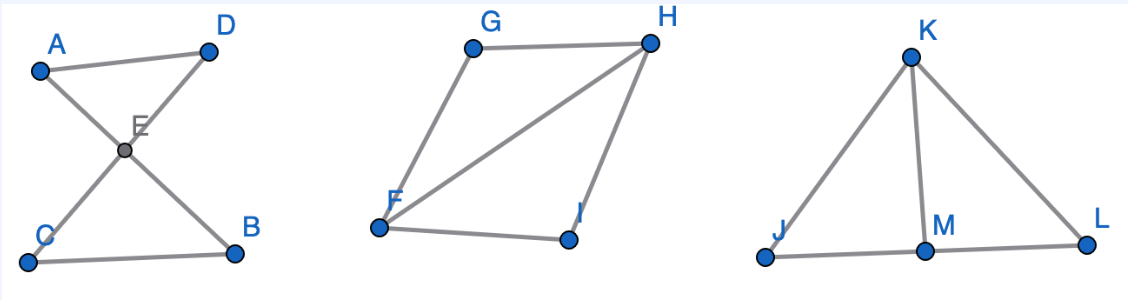
“That distinction — between what a diagram tells you and what it merely suggests — is what makes proof rigorous rather than approximate. Today you are going to build the habit of asking: how do I know that?”

OPENING TASK

What Does the Diagram Tell You? (≈20 min)

Display two or three unmarked diagrams on the board — no tick marks, no arc marks, no labels beyond vertex letters. Include:

- Two triangles formed by two intersecting lines — no marks
- Two triangles sharing a common side — no marks
- Two triangles in a figure where a line appears to bisect something but is not marked



Say: "Look at each diagram. Without being told anything, write down every geometric fact you can state with confidence. For each fact, ask yourself: am I certain this must be true from the diagram alone, or am I assuming it because it looks that way?"

What to listen for and surface

Students should find with confidence: vertical angles at intersections, linear pairs along straight lines, shared/reflexive sides where two triangles share an edge.

Students will want to assume but should not: that a line bisects an angle or side, that two sides are parallel, that two segments are equal. None of these follow from appearance alone.

When a group states something uncertain: 'How do you know that? What in the diagram forces it to be true?' If they cannot cite a geometric reason, it cannot be used in a proof without a given.

Deliberately spend the most time on the unmarked cases. Year 1 data: many students missed shared sides and vertical angles specifically because they were not marked. That is why the diagrams are unmarked. The lesson exists to address this.

NOTES — BUILD ON BOARD BEFORE STUDENTS WRITE (≈10 MIN)

Build each piece on the board collaboratively before students record it. The free information table is the most important — make sure students can distinguish every row.

✓ Free Information — no given required	✗ Must Be Given — cannot assume from appearance
Vertical angles *When two lines intersect, opposite angles are congruent. Follows from the structure of the lines — the vertical angles theorem from Unit 1.*	Parallel lines

Linear pairs *Two angles forming a straight line are supplementary. Always true from diagram structure.*	Congruent segments or angles
Reflexive / shared sides *A segment shared by two triangles is congruent to itself. The reflexive property. Always true.*	Bisected angles or sides
	Midpoints
	Perpendicularity
	Any measurement that is not structurally forced

The logic connection — name this explicitly

Unit 2 established: we cannot use a conditional's consequent without first establishing its antecedent. The same principle applies here. Every statement in a proof requires a justification — except the three types of free information above. Everything else must be earned: from the given, from a postulate, or from a previous step in the proof chain.

A proof is a chain of conditional statements connected by the law of syllogism. The conclusion of one step becomes the antecedent of the next. In a flow chart proof, you can see this shape directly: the output of one box feeds into the input of the next. When you cannot cite a source for a step, the chain is broken.

CPCTC — Full definition, biconditional form, course policy

CPCTC: Corresponding Parts of Congruent Triangles are Congruent.

Biconditional form:

If two triangles are congruent, then all six corresponding parts are congruent (three pairs of sides and three pairs of angles).

AND: If all six corresponding parts of two triangles are congruent, then the triangles are congruent.

Course policy:

Before writing CPCTC as a justification in any proof, board work, quiz, or assessment, write its full meaning: "Corresponding Parts of Congruent Triangles are Congruent: if two triangles are congruent, then all six corresponding parts are congruent." The abbreviation is only permitted after the definition has been written.

Example: If $\triangle ABC \cong \triangle DEF$, CPCTC gives exactly six congruent statements:

Three pairs of congruent sides	Three pairs of congruent angles
AB ≅ DE	∠A ≅ ∠D
BC ≅ EF	∠B ≅ ∠E
AC ≅ DF	∠C ≅ ∠F

⚠️ **Two critical points about CPCTC**

It is not new: CPCTC is the definition of congruence from Lesson 3.2. Students were using it as a label without connecting it to that meaning. The policy forces the connection every time.

Order matters: △ABC ≅ △DEF means A↔D, B↔E, C↔F. Writing △ABC ≅ △EDF is a different claim. Push for correct correspondence notation throughout the poster task.

POSTER TASK — TRIANGLE CONGRUENCE CLASSIFICATION (≈55 MIN)

Activity credit: Mathmedic (mathmedic.com)

Activity link: https://portal.mathmedic.com/lesson-plans/course/Geometry/unit/4/day/12

Print one set of triangle pair cards per group. Groups arrange classified pairs on poster board with marks, shortcut, and congruence statement.

Poster task instructions — read aloud to students

For each pair of triangles: (1) mark all congruent parts you can identify — from the given information and from what the diagram always provides (vertical angles, shared sides); (2) decide which congruence shortcut applies, or state that the triangles cannot be proven congruent; (3) write a correct triangle congruence statement — order matters; (4) arrange your classified pairs on the poster; (5) every group member must be able to explain every pair.

⚠️ **What to watch for during circulation**

Most common error 1: Students miss shared sides and vertical angles because they are not marked. Ask: ‘What does the diagram always give you? Is there a shared side here?’

Most common error 2: Students write congruence statements with incorrect correspondence order. Ask: ‘Read me your statement. What does the order of letters tell me about which vertices correspond?’

Assumption error: A group assumes bisection or parallelism from appearance. Redirect: ‘Where is that given? If it’s not in the free list and it’s not stated, you cannot use it.’

Not enough information: Some pairs in the Mathmedic set are not provably congruent. Students should say ‘not enough information’ and identify what is missing.

Extension for early finishers: Pick one congruent pair and list all six CPCTC statements that follow. This is direct practice for proof writing in Lesson 3.4.

CLOSING CONSOLIDATION (≈5 MIN)

Ask two or three groups to share: one pair they found straightforward, one they found difficult. For each ask:

"What told you those parts were congruent — the given, the diagram, or both? If these triangles are congruent, what does CPCTC allow you to conclude?"

> **What this closes — and what it opens**
>
> *The CPCTC question here is deliberately not answered fully. Proving triangles congruent is not the endpoint — it is the gateway. CPCTC allows us to then prove something further: a specific pair of sides or angles that we could not prove directly. Students will discover this through proof in Lesson 3.4. Leave the question open.*

STUDENTS MAKE NOTES ON

> ✎ **Students make notes on:**
>
> - Free information table: what the diagram always gives vs. what must be given (built collaboratively on board)
> - The proof chain idea: every step requires a justification except free information; conclusion of one step becomes antecedent of the next
> - CPCTC: full definition, biconditional form, correspondence order note, course policy, six-statement example from △ABC ≅ △DEF

Connection Points	• Draws from Unit 1: vertical angles theorem and linear pairs — first proven in Lesson 1.4 — appear as free diagram information • Draws from Unit 2: law of syllogism is the structure of a proof chain; antecedent must be established before consequent can be used • Draws from Lesson 3.2: CPCTC is the definition of congruence, not a new idea • Feeds into Lesson 3.4: proof writing begins with exactly these tools — free information, shortcuts, CPCTC