

Check Your Understanding 7.6

Name: _____ Period: _____ Date: _____

Show all work. Estimate from the class trig table before using a calculator.

Basic	Intermediate	Advanced
Row 3 Spiral — Solving for Missing Angles → <i>Inverting the function</i>		
<i>Basic — Spiral</i> 1. Find each angle to the nearest degree. Estimate first. (a) Right triangle: $\text{opp} = 7$, $\text{hyp} = 10$. (b) Right triangle: $\text{adj} = 5$, $\text{opp} = 9$.	<i>Intermediate — Spiral</i> 2. A zip-line cable stretches from a 14 m platform to an anchor 22 m from the base. Find the angle with horizontal and the cable length.	<i>Advanced — Spiral</i> 3. Observers A and B are 300 m apart on flat ground. Both sight the top T of a cliff. $\angle TAB = 72^\circ$, $\angle TBA = 53^\circ$. Find the height of the cliff.
Row 4 Spiral — Special Right Triangles → <i>Exact values worth deriving</i>		
<i>Basic — Spiral</i> 4. A 45-45-90 triangle has hypotenuse $5\sqrt{2}$. Find both legs exactly. 5. A 30-60-90 triangle has long leg $\sqrt{6}$. Find the short leg and hypotenuse.	<i>Intermediate — Spiral</i> 6. Find the exact diagonal of a rectangle with sides 3 and $3\sqrt{3}$. 7. A regular hexagon has side 4. Find the exact distance between opposite vertices.	<i>Advanced — Spiral</i> 8. Find the exact area of a 30-60-90 triangle with hypotenuse 8, and of a 45-45-90 triangle with hypotenuse 8. Which is larger?
Row 5 Spiral — Estimation → <i>Trig as a function</i>		
<i>Basic — Spiral</i> 9. A calculator gives $\cos 72^\circ \approx 0.309$. Is this reasonable? Use specific benchmarks.	<i>Intermediate — Spiral</i> 10. A student gets $\tan 80^\circ \approx 5.67$. Describe qualitatively what happens to $\tan \theta$ as $\theta \rightarrow 90^\circ$ and explain why this value is reasonable.	<i>Advanced — Spiral</i> 11. Without a calculator, order from least to greatest: $\sin 25^\circ$, $\cos 25^\circ$, $\sin 65^\circ$, $\cos 65^\circ$. Justify fully.

Basic	Intermediate	Advanced
Trig Beyond Right Triangles — Law of Sines → Extending invariants		
<i>Basic</i> 12. In $\triangle ABC$: $A = 44^\circ$, $B = 71^\circ$, $a = 9$ cm. Find side b. 13. In $\triangle PQR$: $P = 58^\circ$, $Q = 63^\circ$, $p = 14$ m. Find side q. 14. Find the area of a triangle with sides 7 cm and 10 cm and included angle 39°.	<i>Intermediate</i> 15. In $\triangle ABC$: $A = 51^\circ$, $a = 12$, $b = 15$. Find angle B. 16. Rangers at A and B (20 km apart) spot fire F. $\angle FAB = 61^\circ$, $\angle FBA = 54^\circ$. Find the distance from each station to the fire? 17. $\triangle ABC$: $A = 48^\circ$, $B = 67^\circ$, $C = 65^\circ$, $a = 11$ cm. Find the area.	<i>Advanced</i> 18. Derive the Law of Sines from the area formula. Show $\text{Area} = \frac{1}{2}bc \cdot \sin A = \frac{1}{2}ac \cdot \sin B = \frac{1}{2}ab \cdot \sin C$ and explain how dividing by $\frac{1}{2}abc$ gives $\sin A/a = \sin B/b = \sin C/c$. 19. Why does the derivation hold for obtuse triangles? Explain why $\sin(180^\circ - \theta) = \sin \theta$ means the ratio is unchanged.
Algebra Spiral Solve factorable quadratics	<i>Algebra Spiral</i> Solve by factoring. (a) $x^2 + 2x - 24 = 0$ (b) $3x^2 - 11x - 20 = 0$ (c) $4x^2 = 20x$	

Answers — CYU 7.6**Row 3 Spiral**

- 1a. $\sin^{-1}(7/10) \approx 44^\circ$.
 1b. $\tan^{-1}(9/5) \approx 61^\circ$.
 2. Angle = $\tan^{-1}(14/22) \approx 32.5^\circ$. Cable = $\sqrt{(196+484)} \approx 26.1$ m.
 3. $\angle ATB = 55^\circ$. $AT = 300 \cdot \sin 53^\circ / \sin 55^\circ \approx 292.9$ m. Height = $292.9 \cdot \sin 72^\circ \approx 278.7$ m.

Row 4 Spiral

4. Legs = 5.
 5. Short = $\sqrt{2}$, hyp = $2\sqrt{2}$.
 6. $\sqrt{(9+27)} = 6$.
 7. 8.
 8. 30-60-90: legs 4 and $4\sqrt{3}$. Area = $8\sqrt{3} \approx 13.86$. 45-45-90: legs $4\sqrt{2}$ each. Area = 16. 45-45-90 is larger.

Row 5 Spiral

9. $\cos 60^\circ = 0.5$, $\cos 45^\circ \approx 0.707$. $\cos 72^\circ < \cos 60^\circ$. $0.309 < 0.5$. ✓ Reasonable.
 10. As $\theta \rightarrow 90^\circ$, $\text{adj} \rightarrow 0$ and $\tan = \text{opp}/\text{adj} \rightarrow \infty$. 5.67 at 80° is consistent.
 11. $\sin 25^\circ = \cos 65^\circ \approx 0.423$. $\cos 25^\circ = \sin 65^\circ \approx 0.906$. Order: $\sin 25^\circ = \cos 65^\circ < \cos 25^\circ = \sin 65^\circ$.

Law of Sines

12. $C = 65^\circ$. $b = 9 \cdot \sin 71^\circ / \sin 44^\circ \approx 12.23$ cm.
 13. $R = 59^\circ$. $q = 14 \cdot \sin 63^\circ / \sin 58^\circ \approx 14.68$ m.
 14. $\frac{1}{2} \cdot 7 \cdot 10 \cdot \sin 39^\circ \approx 22.01$ cm².
 15. $\sin B = 15 \cdot \sin 51^\circ / 12 \approx 0.972$. $B \approx 76.5^\circ$.
 16. $\angle F = 65^\circ$. $AF = 20 \cdot \sin 54^\circ / \sin 65^\circ \approx 17.86$ km. $BF = 20 \cdot \sin 61^\circ / \sin 65^\circ \approx 19.32$ km.
 17. $b \approx 13.62$. Area = $\frac{1}{2} \cdot 11 \cdot 13.62 \cdot \sin 65^\circ \approx 67.8$ cm².
 18. Height from C = $b \cdot \sin A$. Area = $\frac{1}{2}cb \cdot \sin A$. Similarly $\frac{1}{2}ac \cdot \sin B = \frac{1}{2}ab \cdot \sin C$. Divide by $\frac{1}{2}abc$: $\sin A/a = \sin B/b = \sin C/c$. ■
 19. For obtuse A, height = $b \cdot \sin(180^\circ - A) = b \cdot \sin A$ since $\sin(180^\circ - \theta) = \sin \theta$. The area formula is unchanged. ■

Algebra Spiral

- (a) $x = 4$ or $x = -6$
 (b) $x = 5$ or $x = -4/3$
 (c) $x = 0$ or $x = 5$