

Parallelogram Properties: The Engine of the Unit

Unit 5: Quadrilaterals • Day 2 of unit

At a Glance

Pacing: Task: Prove everything you can (~40 min, extensions given individually) • Gallery walk if time allows (~8 min) • Consolidation: the key move + four proofs (~25 min) • Worked example (~8 min) • Notes + CYU is homework

Materials: Boards pre-drawn with parallelogram ABCD and both diagonals before class — one per group. Markers.

Nav instrument rows active: Row 2 (Parallelogram Properties) — all three levels activate today

Pre-draw before class: Parallelogram ABCD with both diagonals drawn and intersection point E labeled on each group's board section. This removes setup time and puts the diagonal in place — which is the key move — without telegraphing it as the starting step.

Property 1 note: Opposite sides congruent was proven in Unit 3 (Lesson 3.5b, Problem 4). Treat it as review and move quickly to the three new properties.

OBJECTIVES

By the end of this lesson students can:

- State all four parallelogram properties as conditional statements.
- Prove all four properties using the hidden triangle strategy.
- Prove the converse: if both pairs of opposite sides are congruent, the quadrilateral is a parallelogram.
- Use the properties and their converses algebraically to find missing sides and angles.

 **Narrative Arc** — Name the strategy before any proof begins

“Every proof in this lesson uses the same move: draw a diagonal to create two triangles, prove the triangles congruent, then extract what you need by CPCTC. The parallelogram is the object being studied — the triangles inside it are the key. This is the big idea of the unit in action.”

BOARD TASK — PROVE EVERYTHING YOU CAN (≈40 MIN)

Prompt — written on boards before class

Board setup

Draw parallelogram ABCD with both diagonals and intersection point E on each group's board section before students arrive. Label the vertices clearly. The diagonals are already drawn — do not tell students why. They will discover the reason when they use them.

Prompt

ABCD is a parallelogram. Both diagonals have been drawn. Mark everything you know is true from the definition. Then prove as many additional facts as you can — about the sides, the angles, and the diagonals. Write each result as a conditional statement. Cite every reason.

⚠ What to watch for — the four properties to reach

Property 1 — Opposite sides congruent: Unit 3 review. Draw diagonal AC. Alternate interior angles on each pair of parallel sides + reflexive diagonal $\rightarrow$ ASA $\rightarrow$ CPCTC. Groups should reproduce this quickly. If they cannot, prompt: 'Which lesson in Unit 3 proved this?'

Property 2 — Opposite angles congruent: Falls out of the same ASA triangles by CPCTC. Or: since consecutive angles are supplementary (Property 3), $\angle A + \angle B = 180^\circ$ and $\angle B + \angle C = 180^\circ$, so $\angle A = \angle C$. Either route is valid.

Property 3 — Consecutive angles supplementary: Co-interior angles on parallel lines (our adaptation of postulate 5). $AB \parallel DC \rightarrow \angle DAB + \angle ABC = 180^\circ$. Direct from the parallel lines work. No triangle needed.

Property 4 — Diagonals bisect each other: Alternate interior angles on both pairs of parallel sides give two angle pairs. Combined with $AB = DC$ (Property 1), ASA on $\triangle ABE$ and $\triangle CDE$ gives $AE = CE$ and $BE = DE$ by CPCTC.

Give the extension prompts individually to groups once they have found each property.

Extension 1 — The Converse (give individually after Property 1)

You have shown: if a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Is the converse true? If both pairs of opposite sides of a quadrilateral are congruent, must it be a parallelogram? If you think so, prove it.

Extension 2 — A Different Converse (give individually after Property 4)

You have shown: the diagonals of a parallelogram bisect each other. Is the converse true? If the diagonals of a quadrilateral bisect each other, must it be a parallelogram? If you think so, prove it.

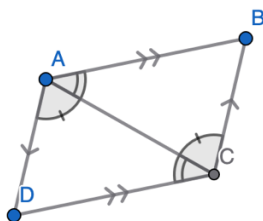
GALLERY WALK (IF TIME ALLOWS — ≈ 8 MIN)

- 'This group found four properties. Which proof used the same triangles as which other proof?'
- 'Every proof on these boards involves triangles. Point to where the triangles are hiding in the original parallelogram.'
- 'This group proved the converse. Walk through the argument — where does the parallel line come from?'

CONSOLIDATION — FOUR PROOFS AND THE CONVERSE

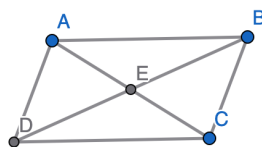
State the key move explicitly before the first proof. Then work through each property. Students have already found them at the boards — consolidation is about precision, citation, and naming.

Property 1 — Opposite Sides Congruent (Unit 3 review) — Proof	
<i>Given: Parallelogram ABCD with diagonal AC drawn. $AB \parallel DC$ and $AD \parallel BC$.</i>	
Statement	Justification
$AB \parallel DC$ and $AD \parallel BC$	Definition of parallelogram.
$\angle BAC = \angle DCA$	Alternate interior angles: $AB \parallel DC$, transversal AC.
$\angle BCA = \angle DAC$	Alternate interior angles: $AD \parallel BC$, transversal AC.
$AC = AC$	Reflexive property.
$\triangle ABC \cong \triangle CDA$	ASA.
$AB = CD$ and $BC = AD$ ■	CPCTC.
<i>Conditional form: If a quadrilateral is a parallelogram, then opposite sides are congruent.</i>	



Property 2 — Opposite Angles Congruent — Proof	
<i>Given: From the Property 1 proof, $\triangle ABC \cong \triangle CDA$ is already established.</i>	
Statement	Justification
$\triangle ABC \cong \triangle CDA$	Established in Property 1.
$\angle ADB = \angle CBA$ ■	CPCTC.
<i>Conditional form: If a quadrilateral is a parallelogram, then opposite angles are congruent. — Note: $\angle DAB = \angle BCD$ follows by the same argument using diagonal BD, or by the supplementary argument below.</i>	

Property 3 — Consecutive Angles Supplementary — Proof	
<i>Given: Parallelogram ABCD. $AB \parallel DC$.</i>	
Statement	Justification
$AB \parallel DC$	Definition of parallelogram.
$\angle DAB + \angle ADC = 180^\circ$	Co-interior angles: $AB \parallel DC$, transversal AD.
$\angle DAB$ and $\angle ADC$ are supplementary ■	Definition of supplementary.
<i>Conditional form: If a quadrilateral is a parallelogram, then consecutive angles are supplementary. — Same argument applies to every consecutive pair: $\angle B + \angle C$, $\angle C + \angle D$, $\angle D + \angle A$.</i>	

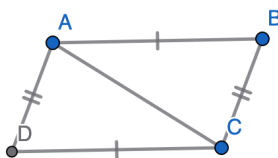


Property 4 — Diagonals Bisect Each Other — Proof

Given: Parallelogram ABCD with both diagonals drawn, intersecting at E.

Statement	Justification
$AB \parallel DC$	Definition of parallelogram.
$\angle ABE = \angle CDE$	Alternate interior angles: $AB \parallel DC$, transversal BD.
$\angle BAE = \angle DCE$	Alternate interior angles: $AB \parallel DC$, transversal AC.
$AB = DC$	Property 1 — opposite sides congruent.
$\triangle ABE \cong \triangle CDE$	ASA.
$AE = CE$ and $BE = DE$ ■	CPCTC — diagonals bisect each other.

Conditional form: If a quadrilateral is a parallelogram, then the diagonals bisect each other.



Converse — If Opposite Sides Congruent, Then Parallelogram — Proof

Given: Quadrilateral ABCD with $AB = CD$ and $BC = AD$. Diagonal AC drawn.

Statement	Justification
$AB = CD$ and $BC = AD$	Given.
$AC = AC$	Reflexive property.
$\triangle ABC \cong \triangle CDA$	SSS.
$\angle BAC = \angle DCA$	CPCTC.
$AB \parallel DC$	Equal alternate interior angles: transversal AC. Converse of alternate interior angles theorem (Unit 3).
$\angle BCA = \angle DAC$	CPCTC.
$BC \parallel AD$	Equal alternate interior angles: transversal AC.
ABCD is a parallelogram ■	Both pairs of opposite sides parallel — definition of parallelogram.

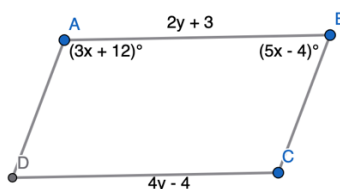
Conditional form: If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.

Properties summary — display before the worked example

Build or display this table after the four proofs. Students confirm their notes against it before the worked example.

Property	Conditional form	Key proof move
Opposite sides congruent	<i>If parallelogram, then $AB = CD$ and $BC = AD$</i>	Diagonal $\rightarrow$ ASA $\rightarrow$ CPCTC
Opposite angles congruent	<i>If parallelogram, then $\angle A = \angle C$ and $\angle B = \angle D$</i>	Same triangles $\rightarrow$ CPCTC
Consecutive angles supplementary	<i>If parallelogram, then $\angle A + \angle B = 180^\circ$</i>	Co-interior angles on parallel lines
Diagonals bisect each other	<i>If parallelogram, then $AE = CE$ and $BE = DE$</i>	Both diagonals $\rightarrow$ ASA $\rightarrow$ CPCTC

WORKED EXAMPLE — WALK THROUGH TOGETHER



In parallelogram ABCD: $\angle A = (3x + 12)^\circ$, $\angle B = (5x - 4)^\circ$, $AB = (2y + 3)$, $CD = (4y - 7)$

Find x , y , and all four angles.

Step 1 — Consecutive angles: $(3x + 12) + (5x - 4) = 180 \rightarrow 8x + 8 = 180 \rightarrow x = 21.5$

$\angle A = 3(21.5) + 12 = 76.5^\circ$ $\angle B = 5(21.5) - 4 = 103.5^\circ$ Check: $76.5 + 103.5 = 180^\circ \checkmark$

Step 2 — Opposite angles: $\angle C = \angle A = 76.5^\circ$ $\angle D = \angle B = 103.5^\circ$

Step 3 — Opposite sides: $2y + 3 = 4y - 7 \rightarrow 10 = 2y \rightarrow y = 5$. $AB = CD = 13$.

STUDENTS MAKE NOTES ON

Students make notes on:

- All four properties: conditional form and key proof move for each (students record from consolidation)
- The converse: conditional form and proof summary
- The hidden triangle strategy named explicitly: draw diagonal $\rightarrow$ prove triangles $\rightarrow$ extract by CPCTC
- Worked example: solution recorded step by step

Narrative Arc — End of consolidation — transition to Lesson 5.3

“Four properties, all proven from two parallel sides. And the converse: four equal sides (in two opposite pairs) force two pairs of parallel sides. We have the engine. Next

lesson we add one more constraint — either four right angles or four equal sides — and ask what new properties emerge. You already have every tool you will need.”

Connection Points

- Draws from Unit 3: alternate interior angles theorem and its converse (Unit 2/3) are the hinge of every proof; CPCTC and ASA/SSS are the tools
- Draws from Lesson 5.1: the parallelogram definition established in Lesson 5.1 is the only given in every proof
- Property 1 draws from Unit 3, Lesson 3.5b (Problem 4) — students proved this previously
- Feeds into Lesson 5.3: all four properties are inherited by rectangles, rhombi, and squares — no re-proving needed
- Feeds into coordinate methods (Lesson 5.5): the converse is the tool for proving parallelogram-ness from coordinates