

Inductive Reasoning

Unit 2: Logic and Reasoning • Day 1 of 11

At a Glance

Pacing: 65 min tasks + gallery walk • 25 min consolidation (ranking, notes, CYU)

Materials: Boards and markers

Nav instrument rows active: Row 1 (Inductive Reasoning) activates today

OBJECTIVES

By the end of this lesson students can:

- Define inductive reasoning.
- Find the n th term of a sequence and extend a visual pattern.
- Explain the fundamental weakness of inductive reasoning: one counterexample disproves a conjecture regardless of how many cases confirmed it.
- Distinguish between ‘the pattern continues’ and ‘we can explain why it must continue.’

NARRATIVE ARC

 **Narrative Arc** — Use at the start of the lesson — and return to it at the end

“We have been reasoning since the first day of this class — noticing patterns, making conjectures, testing them. We have been doing it naturally, because it is how humans reason. Today we are going to look at that process carefully. We are going to name it, test its limits, and then ask: when that kind of reasoning is not enough, what do we do instead?”

“By the end of this unit you will have two reasoning tools instead of one. You will know when each is appropriate, and you will know what makes an argument trustworthy. That is not just a math skill. It is one of the most important things a person can know.”

TASKS

Task 1 — Visual Pattern (adapted from Serra, Discovering Geometry)

Display the visual pattern sequence. Say only: “Work in your groups. Figure out what comes next.”

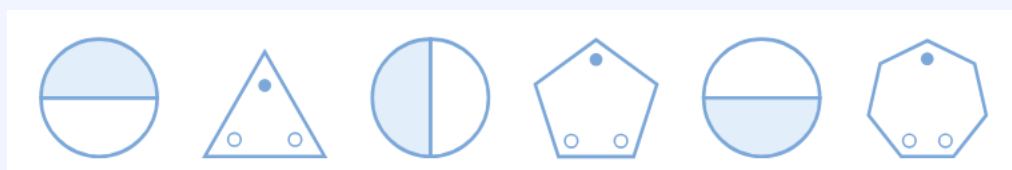


Image from: *Discovering Geometry: An Investigative Approach* by Michael Serra

Once groups are moving, add:

- Draw the next two shapes.
- Draw the 25th shape without drawing all shapes in between.
- Describe the 30th shape. You do not need to draw it.

Teacher note: natural opportunity to reinforce Unit 1 vocabulary. Ask ‘What geometric term describes that shape?’ while circulating — do not stop the task to lecture. Leave at least one group’s board up for consolidation.

Extension ladder: Ask whether there is a formula for the n th shape. Can they describe it algebraically?

Task 2 — The Handshake Problem

Say: “At a meeting, everyone shakes hands with everyone else exactly once. How many handshakes happen if I tell you the number of people in the room?” No further instructions.

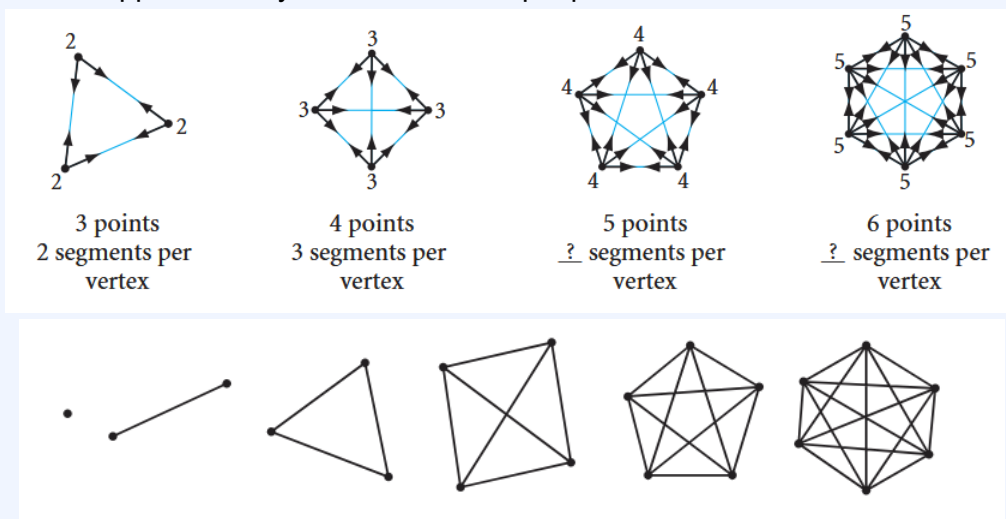


Image from: *Discovering Geometry: An Investigative Approach* by Michael Serra.

Scaffolding (give only what is needed):

- Try a small example. What if there are 2 people? 3?
- Can you draw a picture? Can labeling the people help?
- You might be counting each handshake twice. Does that help?

The key move: after groups find the formula $n(n-1)/2$, ask whether they can explain **why** it must work, not just that it has worked. Groups that connect it to the structure — each of n people shakes $n-1$ hands, each handshake counted twice — have found an explanation, not just a pattern. Name that distinction explicitly.

Extension ladder: Does the formula look familiar? Where have they seen $n(n-1)/2$ before? Can they connect it to the triangular numbers or the staircase area from Unit 0?

Leave at least one group’s board up for consolidation.

Task 3 — Dividing a Circle — The Epistemological Gut-Punch

Say: “Here is my rule: any time there are dots on a circle, you must connect every dot to every other dot. Figure out how many sections the circle gets divided into, depending on the number of dots.”

Number of dots (n)	Sections
1	1
2	2
3	4
4	8
5	16

Expected progression: students build a table and conjecture $S = 2^{(n-1)}$. They will be confident — let them be.

The moment: When most groups have their conjecture, say: ‘Test it for 6 dots.’ The prediction is 32. The actual count is 31. This is not a counting error. Students who recount carefully will still get 31.

! Do not soften this

The pattern was wrong. The failure is the lesson. Students who feel the confidence collapse — who genuinely believed the pattern and then watched it break — have a visceral understanding of inductive reasoning’s fundamental weakness that no amount of explanation could produce.

Answers: True formula: $S = (1/24)(n^4 - 6n^3 + 23n^2 - 18n + 24)$. A degree-4 polynomial students will not encounter until later mathematics. Mention that a correct formula exists; they do not need to find it today.

WHAT TO WATCH FOR

⚠ Watch for

Task 2: Groups who find the formula but cannot explain why it works. This is the most important coaching moment — press for the structural reason, not just the pattern.

Task 3: Groups who recount and still get 31 and suspect their own counting. Reassure them: the count is right. The formula was wrong.

Task 1: Groups who extend the pattern visually but cannot describe the n th term algebraically. Prompt: ‘If I told you $n = 100$, could you describe it without drawing 100 shapes?’

CONSOLIDATION

Gallery walk first. Direct students to boards for Tasks 2 and 3 specifically. No teacher narration during the walk.

After the walk, ask the whole class: all three tasks used the same method — we looked at examples and found a pattern. But are all three results equally trustworthy? Ask students to rank them. Let the class argue. Guide toward:

Most trustworthy

Handshake — some groups can explain why the formula must continue, not just that it has.

Middle

Visual pattern — the rule seems clear but nothing rules out a change at the 7th term.

Least trustworthy

Circle — five clean confirmations were not enough. One counterexample disproves a conjecture regardless of how many cases confirmed it.

State explicitly: this is inductive reasoning. Name it, state its strength (useful for building conjectures), and state its weakness (cannot guarantee a conclusion). End with the question that motivates the whole unit: if we cannot trust patterns alone, what can we trust?

DEFINITIONS AND KEY POINTS

Definition: Inductive Reasoning

The process of observing data, recognizing patterns, and making generalizations.

Strength: Humans are good at recognizing patterns. Powerful for building conjectures.

Weakness: Patterns can be false patterns. Even a pattern that holds for many cases may fail. One counterexample disproves a conjecture — no matter how many cases confirmed it.

Key distinction: Finding a pattern tells us what. Explaining why the pattern must hold is a stronger claim. The handshake formula is the example.

STUDENTS MAKE NOTES ON

Students make notes on:

- Definition of inductive reasoning
- Strength of inductive reasoning (good for conjectures)
- Fundamental weakness (one counterexample is enough to disprove)
- The distinction between ‘the pattern holds’ and ‘we can explain why it must hold’ — using the handshake problem as the example

- The circle task as the demonstration: write the table and the incorrect conjecture, then note where it failed and why

CONNECTION POINTS

Draws from

- Unit 0: specializing, testing, conjecturing — students have been doing inductive reasoning since day 1
- Unit 1: geometric vocabulary for the visual pattern task

Feeds into

- Lesson 2.2a: the failure of inductive reasoning motivates deductive reasoning
- Lesson 2.6: the circle task gut-punch returns when discussing why axioms must be accepted without proof
- Unit 3 onward: ‘failing to find a counterexample does not prove a claim’ is restated in every unit