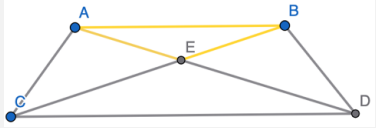


Check Your Understanding 5.4

Show all work. Write CPCTC in full before using it. For proofs: flowchart or paragraph form only.

Basic	Intermediate	Advanced
Row 2 Spiral — Parallelogram Properties → Proving the engine		
<i>Basic — Spiral</i> 1. In parallelogram $MNOP$, $\angle M = (2x + 15)^\circ$ and $\angle N = (3x - 5)^\circ$. Find x and all four angles.	<i>Intermediate — Spiral</i> 2. In parallelogram $RSTU$, diagonal $RT = 24$. The diagonals intersect at V . $SV = (2k + 1)$ and $VU = (4k - 7)$. Find k and the length of diagonal SU . 3. A quadrilateral has vertices $P(-2, 1)$, $Q(2, 3)$, $R(5, -1)$, $S(1, -3)$. Use midpoints to determine whether the diagonals bisect each other.	<i>Advanced — Spiral</i> 4. Given: parallelogram $ABCD$. E is the midpoint of AB and F is the midpoint of CD . Prove: $AEFD$ is a parallelogram.
Row 3 Spiral — Special Parallelograms → Adding constraints, inheriting results		
<i>Basic — Spiral</i> 5. In rectangle $KLMN$, diagonal $KM = (4x + 1)$ and diagonal $LN = (6x - 5)$. Find x and each diagonal. 6. A parallelogram has diagonals that are perpendicular and equal. What type is it? Explain using the hierarchy.	<i>Intermediate — Spiral</i> 7. In rhombus $WXYZ$, diagonal $WY = 20$ and diagonal $XZ = 14$. Find the side length of the rhombus. 8. In a rectangle, the diagonals intersect at P . If one diagonal has length $5x - 3$ and $WP = x + 7$ (where W is one vertex), find x .	<i>Advanced — Spiral</i> 9. Prove that the diagonals of a rhombus bisect the vertex angles.
Trapezoids and Kites → Structure outside the parallelogram family		
<i>Basic</i> 10. In isosceles trapezoid $ABCD$ ($AB \parallel CD$), $\angle D = 58^\circ$. Find $\angle C$, $\angle A$, and $\angle B$. 11. In kite $PQRS$ with $PQ = PS$ and $RQ = RS$, $\angle P = 68^\circ$. Find $\angle QPR$ and $\angle SPR$.	<i>Intermediate</i> 13. Isosceles trapezoid $WXYZ$ has $WX \parallel YZ$. $WZ = 3a + 7$ and $XY = a + 13$. Find a and the length of each diagonal. 14. In kite $ABCD$, $AB = AD = 5$, $CB = CD = 8$, and diagonal $BD = 6$. Find the length of diagonal AC .	<i>Advanced</i> 16. Prove that the diagonals of an isosceles trapezoid are congruent. Use the base angles result as a given step.

Basic	Intermediate	Advanced
12. State the three properties of an isosceles trapezoid as conditional statements.	15. A quadrilateral has vertices A(0, 0), B(3, 4), C(6, 0), D(3, -3). Use slope to show the diagonals are perpendicular. What additional check would confirm it is a kite rather than a rhombus?	17. Given: isosceles trapezoid ABCD with $AB \parallel CD$, $AD = BC$. Diagonals intersect at E. Prove $\triangle ABE$ is isosceles. 
Algebra Spiral <i>Factor</i>	<i>Algebra Spiral</i> Factor each expression completely. (a) $x^2 - 2x - 15$ (b) $4m^2 + 20m$ (c) $2n^2 - 50$	

Answers — CYU 5.4**Row 2 Spiral**

- Consecutive angles supplementary: $(2x+15)+(3x-5)=180 \rightarrow 5x+10=180 \rightarrow x=34$. $\angle M=83^\circ$, $\angle N=97^\circ$, $\angle O=83^\circ$, $\angle P=97^\circ$.
- $SV = VU$: $2k+1=4k-7 \rightarrow k=4$. $SV=VU=9$. $SU=18$.
- Midpoint of $PR = ((-2+5)/2, (1+(-1))/2) = (1.5, 0)$. Midpoint of $QS = ((2+1)/2, (3+(-3))/2) = (1.5, 0)$. Equal midpoints $\rightarrow$ diagonals bisect each other $\rightarrow$ parallelogram.
- $AB \parallel CD$ and $AB = CD$ (parallelogram). E midpoint of $AB \rightarrow AE = AB/2$. F midpoint of $CD \rightarrow DF = DC/2$. $AE = DF$ (halves of equal segments). $AE \parallel DF$ (on parallel lines). One pair of sides equal and parallel $\rightarrow$ $AEFD$ is a parallelogram. ■

Row 3 Spiral

- $4x+1=6x-5 \rightarrow x=3$. Each diagonal = 13.
- Square. Perpendicular diagonals $\rightarrow$ rhombus. Equal diagonals $\rightarrow$ rectangle. Rectangle + rhombus = square.
- Half-diagonals = 10 and 7. Side = $\sqrt{(100+49)} = \sqrt{149}$.
- WP = half diagonal. Full diagonal = $2 \cdot WP = 2(x+7) = 2x+14$. Set equal to $5x-3$: $2x+14=5x-3 \rightarrow 17=3x \rightarrow x=17/3$.
- $AB = AD$ (rhombus) $\rightarrow \triangle ABD$ isosceles $\rightarrow \angle ABP = \angle ADP$ (base angles). $AB = CB \rightarrow \triangle ABC$ isosceles $\rightarrow \angle BAP = \angle BCP$. So diagonal AC bisects $\angle A$; diagonal BD bisects $\angle B$. ■

Trapezoids and Kites

- $\angle C = 58^\circ$. $\angle A = \angle B = 122^\circ$.
- $\angle QPR = \angle SPR = 34^\circ$ (main diagonal bisects $\angle P$).
- (1) If isosceles trapezoid, then base angles congruent. (2) If isosceles trapezoid, then diagonals congruent. (3) If isosceles trapezoid, then co-base angles supplementary.
- $3a+7=a+13 \rightarrow a=3$. Each diagonal = 16.
- Diagonals perpendicular; main diagonal bisects BD . Half $BD=3$. In $\triangle ABE$: $AE=\sqrt{(25-9)}=4$. In $\triangle CBE$: $CE=\sqrt{(64-9)}=\sqrt{55}\approx 7.42$. $AC\approx 11.42$.
- Diagonal AC : slope 0. Diagonal BD : undefined (vertical). Perpendicular. To confirm kite: check two pairs of consecutive equal sides (not all four equal).
- $\angle ADC = \angle BCD$ (base angles). $AD = BC$ (given). $CD = CD$ (reflexive). $\triangle ACD \cong \triangle BDC$ (SAS). $AC = BD$ (CPCTC). ■
- $AC = BD$ (diagonals congruent, from Q16 proof). $\triangle ACD \cong \triangle BDC$ (SAS as above). $AE = BE$ (CPCTC on the intersection segments). $\triangle ABE$ has $AE = BE \rightarrow$ isosceles. ■

Algebra Spiral

- $(x-5)(x+3)$
- $4m(m+5)$
- $2(n-5)(n+5)$