

The Named Argument Forms

Unit 2: Logic and Reasoning • Day 7 of 11 • Law of Syllogism Discovered • All Four Valid Forms Named

At a Glance

Pacing: Thinking task + gallery walk (40 min) • Guided notes: four valid forms (25 min) • CYU begins (25 min)

Materials: Boards and markers

Nav instrument rows active: Rows 1–4 spiral • Row 5 (Named Argument Forms) activates today

OBJECTIVES

By the end of Days 1 and 2 students can:

- Apply the law of detachment, law of syllogism, denying the consequent, and disjunctive syllogism.
- Construct a multi-step chain of conditionals using the law of syllogism.
- Explain why the law of syllogism is the engine of all deductive reasoning.
- Solve a Lewis Carroll puzzle by chaining conditionals and applying contrapositives.

THINKING TASK

Can You Get There in One Step? (≈30 min)

Part 1

If it is raining → the game is cancelled. If the game is cancelled → we go to the mall. If we go to the mall → we spend money.

If it is raining, what can you conclude? Write every step. Write the chain symbolically using $p \rightarrow q$ notation. Can you state the final conclusion in one step? What does that one step look like? Is there a pattern?

Part 2

If it is raining → the game is cancelled. If we go to the mall → we spend money.

If it is raining, what can you conclude? Why is this harder? What is missing?

⚠ The broken chain in Part 2 is the key moment

Groups that get stuck will feel exactly why the middle term matters — the consequent of the first premise must match the antecedent of the second. That frustration is the intuitive understanding of the law of syllogism before it is named. Do not rescue them too quickly.

Extension ladder: Build your own three-step chain starting from a conclusion and working backwards. Then attempt the Lewis Carroll poultry puzzle (see Day 2).

Gallery walk: spotlight (1) board with the three-step chain written symbolically, (2) board showing the one-step collapse, (3) board explaining what is missing in Part 2. Move from least to most complete.

CONSOLIDATION — FOUR VALID FORMS

Teacher-Guided Notes | The Four Valid Argument Forms (≈ 25 minutes)

Build the table together. Start with the law of syllogism — students just discovered it. For each form, ask: "Which of the four conditional forms is this using?" This reinforces the connection from 2.3 and 2.2b.

Name	Symbolic Form	Example	Conditional Form Used
Law of Syllogism ✓	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	<i>Rain $\rightarrow$ cancelled.</i> <i>Cancelled $\rightarrow$ mall. $\therefore$ Rain $\rightarrow$ mall.</i>	Chains two conditionals. The consequent of the first must be the antecedent of the second. This is the engine of all deductive reasoning.
Law of Detachment ✓	$p \rightarrow q$ p $\therefore q$	<i>If it rains $\rightarrow$ game cancelled. It is raining. $\therefore$ Game is cancelled.</i>	Uses the conditional $p \rightarrow q$ directly. p is true, so q must follow.
Denying the Consequent ✓	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	<i>If it rains $\rightarrow$ game cancelled. Game is not cancelled. $\therefore$ It is not raining.</i>	Uses the contrapositive $\sim q \rightarrow \sim p$. Valid because the contrapositive is equivalent to the original.
Disjunctive Syllogism ✓	$p \vee q$ $\sim p$ $\therefore q$	<i>It is raining or snowing. It is not raining. $\therefore$ It is snowing.</i>	Either/or premise with one option eliminated. The other must be true. <i>Requires exactly two options.</i>

If You Give a Mouse a Cookie — say this after naming the law of syllogism

The book by Laura Numeroff is a 14-step application of the law of syllogism. If you give a mouse a cookie $\rightarrow$ he'll want a glass of milk $\rightarrow$... $\rightarrow$ he'll want another cookie. The whole story collapses to: if you give a mouse a cookie, you'll end up giving him a second cookie.

Then say: Geometry works the same way. Except instead of starting with one cookie, we start with 10 premises — 5 postulates and 5 common notions — and everything we prove traces back through chains of conditional reasoning. The law of syllogism is the mechanism that makes every step possible.

 Students make notes on:

- Four valid argument forms: law of syllogism, law of detachment, denying the consequent, disjunctive syllogism — with symbolic form, example, and which conditional form each uses (student sheet provided)
- Key insight fill-in: the valid forms use the conditional and its contrapositive; the fallacies use the converse and inverse

**Connection
Points**

- Feeds directly into Lesson 2.5: the law of syllogism is how geometry works — 10 premises, 465 theorems, all connected by conditional chains
- Extension preview: Lewis Carroll puzzle (resolved Day 2) requires both the law of syllogism and the contrapositive together