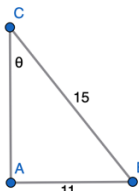


Check Your Understanding 7.5

Name: _____ Period: _____ Date: _____

Show all work. Estimate from the class trig table before using a calculator.

Basic	Intermediate	Advanced
Row 2 Spiral — Solving for Missing Sides → Using invariants		
<i>Basic — Spiral</i> 1. Find each missing value. Estimate first. (a) Right triangle: 33° angle, $\text{adj} = 14$. Find hypotenuse. (b) Right triangle: 71° angle, $\text{hyp} = 9$. Find opposite side.	<i>Intermediate — Spiral</i> 2. From the base of a building, the angle of elevation to the top of an adjacent tower is 48° . From the top of the building (25 m high), the angle is 22° . Find the tower height.	<i>Advanced — Spiral</i> 3. A ship is 12 km due south of a lighthouse. A second ship is 9 km due east of the same lighthouse. Find the distance between the two ships and the angle that the line connecting them makes with the north-south line, measured at the first ship's position.
Row 3 Spiral — Solving for Missing Angles → Inverting the function		
<i>Basic — Spiral</i> 4. Find each angle to the nearest degree.  (a) (b) Right triangle: $\text{adj} = 6$, $\text{opp} = 7$.	<i>Intermediate — Spiral</i> 5. A slide is 4.5 m long, platform 2.8 m high. Find the angle with the ground and horizontal distance from base to end.	<i>Advanced — Spiral</i> 6. A surveyor walks 200 m horizontally from a hill base. She observes the top at 19° and a rock formation at 11° . How much higher is the hilltop than the rock?
Row 4 Spiral — Special Right Triangles → Exact values worth deriving		
<i>Basic — Spiral</i> 7. Find the exact value of each expression. (a) $\sin 45^\circ \times \cos 45^\circ$ (b) $\tan 30^\circ \times \tan 60^\circ$ (c) $\sin 60^\circ \div \cos 30^\circ$	<i>Intermediate — Spiral</i> 8. A right triangle has a 30° angle and hypotenuse $4\sqrt{3}$. Find all sides exactly. 9. Find the exact area of an equilateral triangle with side 10.	<i>Advanced — Spiral</i> 10. Using only exact values and geometric reasoning, prove $\tan 30^\circ = 1/\sqrt{3}$. Then rationalize and explain why $1/\sqrt{3} = \sqrt{3}/3$.

Basic	Intermediate	Advanced
Estimation → Trig as a function		
<i>Basic</i> 11. Without a calculator, is each value closer to 0.5 or 0.707? Explain. (a) $\sin 40^\circ$ (b) $\cos 55^\circ$ 12. Does sine increase or decrease from 0° to 90°? Does cosine? 13. At what angle are $\sin \theta$ and $\cos \theta$ equal? What triangle is this?	<i>Intermediate</i> 14. Sketch $y = \sin \theta$ and $y = \cos \theta$ on the same axes, 0° to 90°. Label the crossing point and endpoint values. 15. Explain geometrically — using a right triangle with hyp = 1 — why sine increases and cosine decreases as θ increases.	<i>Advanced</i> 16. A student's calculator gives $\sin 68^\circ \approx 0.927$. Without a calculator, evaluate whether this is reasonable. Identify exact benchmarks, state the direction of change, and explain your conclusion. 17. The class trig table was built from measurements. The graph today was built from exact values. Explain why they should agree and what discrepancies reveal.
Algebra Spiral Solve systems of two equations	<i>Algebra Spiral</i> Solve each system. Show your method. (a) $3x + 2y = 11$ and $x - y = 2$ (b) $y = 4x - 1$ and $2x + 3y = 15$	

Answers — CYU 7.5**Row 2 Spiral**

1a. $14/\cos 33^\circ \approx 16.69$.

1b. $9 \sin 71^\circ \approx 8.51$.

2. Let h = tower height and d = horizontal distance between the building and the tower. From the base of the building: $\tan 48^\circ = h/d$, so $h = d \cdot \tan 48^\circ$. From the top of the building: $\tan 22^\circ = (h - 25)/d$, so $h - 25 = d \cdot \tan 22^\circ$. Substitute $h = d \cdot \tan 48^\circ$ into the second equation:

$$d \cdot \tan 48^\circ - 25 = d \cdot \tan 22^\circ$$

$$d \cdot \tan 48^\circ - d \cdot \tan 22^\circ = 25$$

$$d(\tan 48^\circ - \tan 22^\circ) = 25$$

$$d = 25 / (\tan 48^\circ - \tan 22^\circ) = 25 / (1.1106 - 0.4040) = 25 / 0.7066 \approx 35.4 \text{ m}$$

$$h = 35.4 \times \tan 48^\circ \approx 35.4 \times 1.1106 \approx 39.3 \text{ m. The tower is approximately 39.3 m tall.}$$

3. Distance = $\sqrt{(144+81)} = 15 \text{ km}$. Bearing from ship 1: $\arctan(9/12) \approx 36.9^\circ$ east of north $\approx 037^\circ$.

Row 3 Spiral

4a. $\sin^{-1}(11/15) \approx 47^\circ$.

4b. $\tan^{-1}(7/6) \approx 49^\circ$.

5. Angle = $\sin^{-1}(2.8/4.5) \approx 38.5^\circ$. Horizontal = $\sqrt{(4.5^2 - 2.8^2)} \approx 3.52 \text{ m}$.

6. Top = $200 \tan 19^\circ \approx 68.9 \text{ m}$. Rock = $200 \tan 11^\circ \approx 38.9 \text{ m}$. Diff $\approx 30.0 \text{ m}$.

Row 4 Spiral

7a. $1/2$

7b. 1

7c. 1

8. Short = $2\sqrt{3}$, long = 6.

9. $25\sqrt{3}$.

10. 30-60-90 hyp 2: opp = 1, adj = $\sqrt{3}$. $\tan 30^\circ = 1/\sqrt{3}$. Rationalize: $\sqrt{3}/3$.

Estimation11a. Closer to 0.5 (between $\sin 30^\circ = 0.5$ and $\sin 45^\circ \approx 0.707$, closer to 30°).11b. $\cos 55^\circ = \sin 35^\circ$. Between 0.5 and 0.707, closer to 0.5.

12. Sine increases. Cosine decreases.

13. 45° . 45-45-90 — both legs equal.14. Accept: sine $0 \rightarrow 1$, cosine $1 \rightarrow 0$, crossing at $(45^\circ, \sqrt{2}/2 \approx 0.707)$.15. Hyp = 1. As θ grows the opposite leg ($= \sin \theta$) lengthens and the adjacent leg ($= \cos \theta$) shortens — they trade length while the hypotenuse stays fixed.16. $\sin 60^\circ \approx 0.866$, $\sin 90^\circ = 1$. 68° is between, closer to 60° . Value should be between 0.866 and 1. 0.927 fits. Reasonable.

17. Same function — one measured, one exact. Should agree within measurement error. Discrepancies reveal Lesson 7.1 measurement imprecision.

Algebra Spiral

(a) $x = 3$, $y = 1$.

(b) $x = 1.5$, $y = 5$.