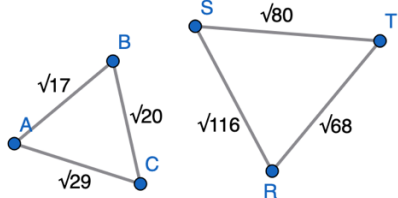


Check Your Understanding 6.5

Show all work. Before applying any property of similar triangles, first justify that the triangles are similar.

Basic	Intermediate	Advanced
Row 2 Spiral — Dilation and Similarity Defined → From experience to definition		
Basic — Spiral 1. A dilation maps $\triangle ABC$ onto $\triangle A'B'C'$ with scale factor $5/2$. $AB = 4$, $BC = 6$. (a) Find $A'B'$ and $B'C'$. (b) What happens to all angle measures?	Intermediate — Spiral 2. Two figures are similar with scale factor $3:7$. (a) A side of the smaller figure is 9. Find the corresponding side of the larger. (b) Is the dilation an enlargement or reduction? How do you know? (c) Could the center of dilation lie inside one of the figures? Explain.	Advanced — Spiral 3. Prove or disprove: all equilateral triangles are similar.
Row 3 Spiral — Triangle Similarity Shortcuts → Proving similarity		
Basic — Spiral 4. $\triangle ABC \sim \triangle RTS$ (vertices in this order). $\angle A = 52^\circ$, $\angle B = 78^\circ$. $AB = 6$, $RT = 9$. (a) Find $\angle R$, $\angle T$, and $\angle S$. (b) Find the scale factor from $\triangle ABC$ to $\triangle RTS$.	Intermediate — Spiral 5. In the figure, $\triangle ABC \sim \triangle RTS$ by SSS~.  (a) Verify the similarity by showing all three ratios are equal. (b) What is the scale factor? What is the ratio of corresponding perimeters?	Advanced — Spiral 6. Given: $\triangle ABC$ with $DE \parallel BC$, where D is on AB and E is on AC. You proved in CYU 6.4 that $AE/AC = DE/BC$. Now use this result to prove that $AD/AB = DE/BC$ as well. Explain what this tells us about the ratio of the segment from the apex to the parallel line versus the full side.

Basic	Intermediate	Advanced
Row 4 Spiral — Indirect Measurement → Applying similarity		
<i>Basic — Spiral</i> 7. At 3 pm, a 6-foot person casts an 8-foot shadow. At the same time, a building casts a 120-foot shadow. How tall is the building? 8. State the assumptions that must hold for the shadow method to produce valid similar triangles.	<i>Intermediate — Spiral</i> 9. Kristen uses a string and ruler method to measure her classroom wall. She tacks a string to the base of the wall and holds the other end to her eye. She adjusts until the bottom of a ruler aligns with the base of the wall and the top with the top of the wall. Draw the situation. <i>Explain why the triangles are similar. If her string length is 3 m and her ruler measures 25 cm, what is the wall height? Show the proportion.</i>	<i>Advanced — Spiral</i> 10. A student uses the mirror method to measure a tree. They place a mirror on the ground 12 m from the base of the tree. They walk back until they see the top of the tree in the mirror. They stop when they are 1.5 m from the mirror. Their eye height is 1.6 m. (a) Draw and label the diagram with these measurements. (b) Calculate the tree height. (c) Identify the source of error most likely to affect the accuracy of this measurement and explain its geometric effect.
Scaling Laws → Similarity in different dimensions		
<i>Basic</i> 11. A triangle has perimeter 10 and area 15. It is scaled by a factor of 4. (a) What is the new perimeter? (b) What is the new area? 12. A rectangular prism has volume 12 ft^3 . It is dilated by a factor of 3. (a) What is the new volume? (b) State the general rules: how do perimeter, area, and volume scale when a figure is scaled by factor k ?	<i>Intermediate</i> 13. Two similar cylinders have radii in the ratio 2:5. (a) What is the ratio of their lateral surface areas? (b) What is the ratio of their volumes? (c) The smaller cylinder has volume $16\pi \text{ cm}^3$. Find the volume of the larger. 14. The area of a smaller similar figure is 36 cm^2 . The area of the larger is 225 cm^2 . Find the scale factor and the ratio of their perimeters.	<i>Advanced</i> 15. Cells rely on surface area to exchange materials their volume needs. (a) Explain why increasing cell size creates a biological problem. (b) Based on your reasoning, explain why most cells must be small. (c) Giant insects existed approximately 300 million years ago (dragonflies with 2-foot wingspans). Using the square-cube law, what does this suggest about the oxygen levels in the atmosphere at that time? Explain your reasoning.
Algebra Spira Factor	<i>Algebra Spiral</i> Factor each expression completely. (a) $x^2 + 3x - 28$ (b) $5m^2 - 30m$ (c) $3n^2 - 48$	

Answers — CYU 6.5

1a. $A'B' = 10$, $B'C' = 15$.

1b. All angle measures are preserved — unchanged.

2a. $9 \times (7/3) = 21$.

2b. Enlargement — scale factor > 1 from smaller to larger.

2c. Yes — the center can lie inside either figure, outside both, or on a vertex. Its position is determined by where the lines through corresponding vertices intersect.

3. True. All equilateral triangles have three 60° angles $\rightarrow$ AA similarity applies to any two equilateral triangles.

4a. $\angle R = 52^\circ$, $\angle T = 78^\circ$, $\angle S = 50^\circ$.

4b. $9/6 = 3/2$.

5a. $2\sqrt{17}/\sqrt{17} = 2\sqrt{20}/\sqrt{20} = 2\sqrt{29}/\sqrt{29} = 2$. All ratios equal 2. SSS~ confirmed.

5b. Scale factor = 2. Ratio of perimeters = 2 (same as scale factor).

6. $\triangle ADE \sim \triangle ABC \rightarrow AD/AB = AE/AC = DE/BC$. The first equality gives $AD/AB = DE/BC$ directly. This tells us the segment from the apex to the parallel line is to the full side as DE is to BC — the same ratio as the sides of the parallel segment.

7. $6/8 = h/120 \rightarrow h = 90$ feet.

8. Both objects perpendicular to the ground; sun's rays parallel (sun effectively at infinity).

9. The string to the ruler forms a small triangle (ruler height / string length) and the wall to the floor forms a large triangle (wall height / floor distance). Both share the angle at the eye. AA similarity. Wall height = (ruler height / string length) $\times$ (floor distance to wall). With $0.25/3 \times \text{room_depth}$: accept any consistent values.

10b. Tree height = $1.6 \times (12/1.5) = 12.8$ m.

10c. Most likely: inaccuracy in measuring the mirror-to-tree distance. Geometric effect: this distance appears in the proportion as the longer leg, so a small error there produces a larger absolute error in the calculated height.

11a. 40.

11b. $15 \times 4^2 = 240$.

12a. $12 \times 3^3 = 12 \times 27 = 324$ ft³.

12b. Perimeter scales by k . Area scales by k^2 . Volume scales by k^3 .

13a. $(2/5)^2 = 4/25$.

13b. $(2/5)^3 = 8/125$.

13c. $16\pi \times (125/8) = 250\pi$ cm³.

14. Area ratio = $225/36 = 25/4$. Scale factor = $\sqrt{(25/4)} = 5/2$. Perimeter ratio = $5/2$.

15a. Volume grows by k^3 but surface area only by k^2 . The surface-area-to-volume ratio decreases as k increases, so a larger cell has proportionally less surface area to supply its interior.

15b. When a cell exceeds a certain size the surface-area-to-volume ratio becomes too small to diffuse nutrients fast enough. Cells divide rather than grow indefinitely.

15c. Giant insects would have needed a higher surface-area-to-volume ratio for respiration (insect respiration is passive, via diffusion through the cuticle). A richer oxygen atmosphere would allow diffusion across a smaller relative surface area — supporting larger bodies. Evidence suggests atmospheric oxygen was $\sim 35\%$ (vs 21% today).

(a) $(x + 7)(x - 4)$

(b) $5m(m - 6)$

(c) $3(n - 4)(n + 4)$