

Polygons and Area in the Plane

Unit 1: Foundations of Geometry • Day 4 of unit

At a Glance

Pacing: ~45 min tasks (security camera + triangle in rectangle) • ~15 min notes • ~30 min CYU begins in class

Materials: Boards and markers • Compasses (1 per group — for the slanted-side demonstration) • Security camera task image (Illustrative Mathematics, link below)

Nav instrument rows active: Rows 1–3 spiral • Row 4 (Polygons and Area) activates today

Task timing warning: Do not let the task run past 45 minutes. The consolidation and practice time are necessary.

OBJECTIVES

By the end of this lesson students:

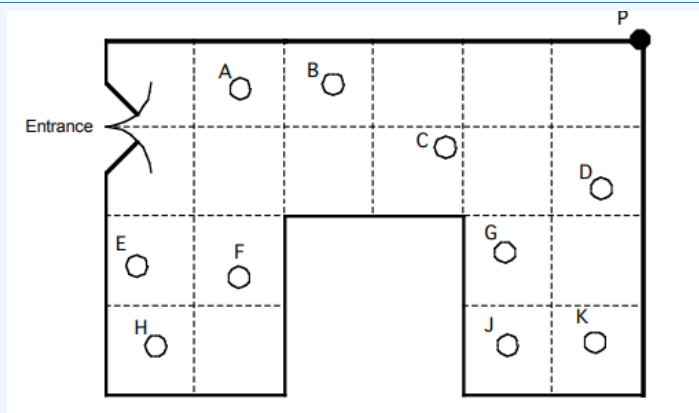
- Can define polygon, convex, and concave, and classify polygons by number of sides.
- Can find the perimeter of a polygon with slanted sides using the distance formula.
- Can find the area of rectangles, triangles, and compound figures in the coordinate plane.
- Can explain why the area of a triangle is half the area of its enclosing rectangle using a visual argument.

TASKS

Task 1 — The Security Camera (~25 min)

Source

Illustrative Mathematics: tasks.illustrativemathematics.org/content-standards/tasks/115



A shop owner wants to prevent shoplifting. A security camera is mounted at point P and can rotate 360°. People in the store are labeled points.

- Which people cannot be seen by the camera, and how do you know?
- The owner believes the camera cannot see 15% of the store. Is he right? Convince me.

- What is the best location for the camera to maximize coverage?
- Draw some store shapes where a corner camera sees the whole store. Draw some where it cannot. What is different about them?

⚠ What to look for during the task

The shapes on the boards will include convex and concave polygons naturally. The consolidation question — what is different about the shapes where the corner camera works versus where it doesn't? — leads directly to the definition of convexity. Let students name the distinction in their own words before giving them the term.

Task 2 — Triangle in the Rectangle (≈20 min)

Each group draws a rectangle. Someone places a point anywhere on the top side, then draws lines from that point to both bottom corners. What fraction of the rectangle does the triangle take up?

- Play, conjecture, test — does the fraction change if you move the point?
- Can you find a visual argument for why it must always be true?

The visual argument to reach

Draw a vertical from the top point to the base. You now see two smaller rectangles, each divided exactly in half by one of the triangle's sides. The triangle is exactly half the original rectangle. Therefore $A = \frac{1}{2}b \times h$.

Push for this argument. Students who can see why $A = \frac{1}{2}bh$ rather than just applying it are students who will not forget it.

Extension: Apply the same reasoning to a parallelogram. What is its area formula? Can you see it as a rearrangement of a rectangle?

CONSOLIDATION — DEFINITIONS, DISTANCE FORMULA, AND COMPASS DEMO

The slanted-side demonstration — use a compass

This is the first appearance of a compass in the course. Students will see it again in Unit 3. Use it here to show why counting grid squares fails for slanted sides: place the compass at two endpoints of a slanted segment and at two endpoints of a horizontal segment that looks visually similar in length. Students will see the arcs do not match. The distance formula is needed.

Say explicitly: this tool will come back in Unit 3 when we do constructions. For now it is showing us something counting cannot.

 Students make notes on:

- Polygon definition; convex and concave definitions with examples; polygon naming table (triangle through decagon)
- Why slanted-side perimeter requires the distance formula rather than counting grid squares
- Area formulas: rectangle ($l \times w$) and triangle ($\frac{1}{2}b \times h$) with visual derivation

CONSOLIDATION REFERENCE

After the tasks, consolidation covers three things in order: (1) polygon definitions and naming from the boards, (2) why slanted-side perimeter requires the distance formula, (3) area formulas with visual derivation. The polygon naming table should be built with the class — students often know triangle through hexagon and need prompting for the rest. Have them fill in the student note sheet as the table is built.

Polygon Naming Table — build with the class

3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon
n	n-gon

Key definitions to confirm students have

Polygon: A closed plane figure formed by three or more line segments (sides) that meet only at their endpoints (vertices), with no two consecutive sides being collinear.

Convex polygon: A polygon where every interior angle measures less than 180° . Equivalently: any line segment connecting two interior points stays entirely inside the polygon.

Concave polygon: A polygon with at least one interior angle greater than 180° (a reflex angle). Also called non-convex. At least one 'dent' is visible.

Connection Points

- Draws from Lesson 1.1: classification and differentiation applied to polygons (same structure as the shoe task)
- Draws from Lesson 1.2: distance formula applied to slanted sides
- Feeds into Unit 5: parallelogram and quadrilateral area arguments use the same visual rectangle reasoning