Unit 4: Triangle Theorems | Lesson 4.5 — Practice

All Four Navigation Instrument Rows

Practice Set 4.5
Show all work. Proofs: flow chart or paragraph.

How to use this practice set:
Work individually. Start at the level matching your navigation instrument self-assessment. Attempt at least that level and try the next level up. For each problem, mark on your navigation instrument: ✓ correct · S silly mistake · H correct with help · G correct in a group · ⊠ wrong · N not attempted. For all proof questions: write the full definition of CPCTC before using the abbreviation.

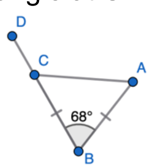
Basic	Intermediate	Advanced

Row 1 — Triangle and Polygon Angle Relationships → *Proving the laws*

Basic

1. State the triangle angle sum theorem as a conditional statement.

2. In triangle ABC, AB = BC and ∠ABC = 68°. Find ∠BAC, ∠BCA, and the exterior angle at C.



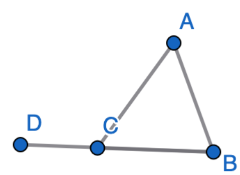
Intermediate

3. An exterior angle of a triangle measures (4x − 3)°. The two non-adjacent interior angles measure (x + 15)° and (2x − 8)°. Find x and all three interior angles of the triangle.

4. Find the exterior angle at one vertex of a regular pentagon. Show all work.

Advanced

5. Given: Triangle ABC with side BC extended to D. Prove that ∠ACD = ∠A + ∠B.
Use the triangle angle sum theorem. Write a complete paragraph or flowchart proof.



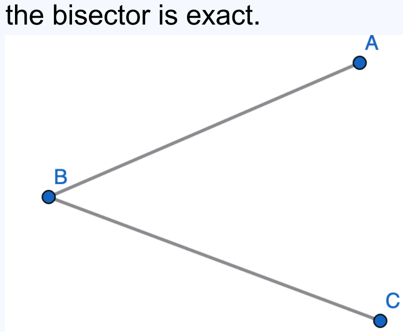
Row 2 — Constructions → *Building the tools*

Basic

6.
Describe the steps to construct a copy of angle ∠ABC on a separate ray using compass and straightedge. For each step, state the postulate or proposition that allows it.

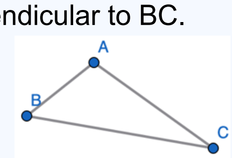
Intermediate

7.
Construct the angle bisector of ∠ABC. Show all arc marks clearly. Then identify the triangle congruence that proves the bisector is exact.



Advanced

8.
Construct the altitude from vertex A of the triangle below. Then prove that the segment you constructed is perpendicular to BC.

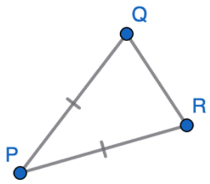


Step	What I did	What allows it
1		
2		
3		
4		

Write the proof below:

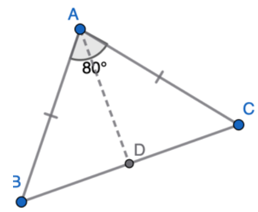
Row 3 — Base Angles Theorem → *Proving and earning the converse*

9. In triangle PQR, PQ = PR. Name the two angles that must be congruent and explain why.



10. In triangle ABC, AB = AC. The base angles satisfy ∠ABC = $(3k + 4)°$ and ∠ACB = $(5k − 18)°$. Find k and all three angles of the triangle.

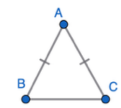
11. In the diagram, △ABC is isosceles with AB = AC. The apex angle ∠BAC = 80°. The angle bisector from A meets BC at D, and BD = $(3x + 1)$ and DC = $(5x − 7)$. Find x and both BD and DC.



12. △XYZ is isosceles with XY = XZ. The exterior angle at Y measures $(7m + 4)°$ and the apex angle ∠YXZ = $(2m + 16)°$. Find m and all interior angles.

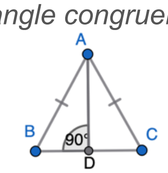
13. Given: Triangle ABC with AB = AC. Prove: ∠ABC ≅ ∠ACB.

State every step. Cite every reason. Name the construction you use.



14. Given: Triangle ABC with AB = AC. Point D is on BC such that AD ⊥ BC. Prove: BD = DC.

Do not use the perpendicular bisector theorem. Prove from triangle congruence.



Row 4 — Bisector Theorems → *Turning constructions into proofs*

15. State the angle bisector theorem as a conditional statement.

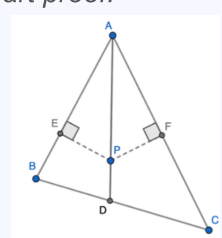
16. Point P is equidistant from both sides of ∠XYZ, with perpendicular distance 7 from each side. What can you conclude about where P lies? State the theorem.

17. Point P lies on the angle bisector of ∠LMN. The perpendicular distance from P to side ML is $(2y + 3)$ and to side MN is $(4y − 9)$. Find y and the perpendicular distance.

18. Point P lies on the perpendicular bisector of segment AB. PA = $(3m + 1)$ and PB = $(5m − 9)$. Find m and PA.

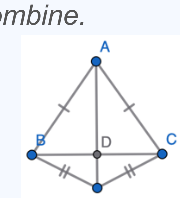
19. Given: In triangle ABC, AD is the angle bisector from A to BC. Point P lies on AD. Prove: P is equidistant from sides AB and AC.

Write a complete paragraph or flowchart proof.



20. Given: Triangles ABC and A′BC share base BC. AB = AC and A′B = A′C. Prove: The line through A and A′ is the perpendicular bisector of BC.

Hint: think about what each isosceles condition tells you about A and A′ separately, then combine.



Answers — Practice Set 4.5

Q1. If a polygon is a triangle, then the sum of its interior angles is 180°.
Q2. AB = BC → ∠BAC = ∠BCA (base angles theorem). 2∠BAC + 68° = 180° → ∠BAC = ∠BCA = 56°. Exterior angle at C = 180° − 56° = 124°.
Q3. $4x - 3 = (x + 15) + (2x - 8) \rightarrow 4x - 3 = 3x + 7 \rightarrow x = 10$. Exterior angle = 37°. Non-adjacent angles: 25° and 12°. Check: 25 + 12 = 37 ✓. Third interior angle = 180 − 25 − 12 = 143°.
Q4. Interior angle sum of a pentagon = 180(5 − 2) = 540°. Each interior angle of a regular pentagon = 540 ÷ 5 = 108°. Exterior angle = 180° − 108° = 72°.
Q5. ∠ACB + ∠ACD = 180° (linear pair). ∠A + ∠B + ∠ACB = 180° (triangle angle sum theorem). Both equal 180° → ∠ACB + ∠ACD = ∠A + ∠B + ∠ACB. Subtract ∠ACB from both sides (Common Notion 3) → ∠ACD = ∠A + ∠B. ■
Q6. Steps: (1) Compass at vertex, draw arc crossing both rays — Postulate 3. (2) Transfer same radius to new ray, label B′ — Postulate 3 / Prop 2–3. (3) Set compass to chord length, mark C′ from B′ — Prop 2–3. (4) Draw ray through C′ — Postulate 1. Proof: SSS on the two triangles → CPCTC gives equal angles.
Q7. Construction: compass at B, arc crossing both sides at B′ and C′ → equal arcs from B′ and C′ crossing at D → draw BD. Congruence: SSS (− AB′ = BC′, B′D = C′D, BD = BD) → CPCTC gives ∠ABD = ∠CBD.
Q8. Construction: compass at A, arc crossing BC at P and Q → equal arcs from P and Q meeting at R → draw AR to H. Proof: AP = AQ, PR = QR, AR = AR → △APR ≅ △AQR (SSS) → ∠PAR = ∠QAR (CPCTC) → AP = AQ, ∠PAH = ∠QAH, AH = AH → △APH ≅ △AQH (SAS) → ∠AHP = ∠AHQ (CPCTC) → linear pair → each 90° → AH ⊥ BC. ■
Q9. ∠PQR = ∠PRQ (base angles theorem: PQ = PR). Both angles are opposite the equal sides.
Q10. AB = AC → ∠ABC = ∠ACB (base angles theorem). So $3k + 4 = 5k - 18 \rightarrow 22 = 2k \rightarrow k = 11$. Each base angle = 37°. ∠BAC = 180° − 74° = 106°. Check: 106 + 37 + 37 = 180 ✓.
Q11. ∠BAC = 80° so base angles = (180 − 80)/2 = 50° each. AD bisects ∠BAC → ∠BAD = ∠CAD = 40°. In △ABD: ∠ADB = 180 − 50 − 40 = 90°. The SAS proof of the base angles theorem gives BD = DC (CPCTC). So $3x + 1 = 5x - 7 \rightarrow x = 4$ → BD = DC = 13.
Q12. XY = XZ → ∠Y = ∠Z (base angles). Exterior angle theorem: 7m + 4 = ∠X + ∠Z. ∠Y = ∠Z = (180 − (2m+16))/2 = 82 − m. So $7m + 4 = (2m + 16) + (82 - m) \rightarrow 7m + 4 = m + 98 \rightarrow 6m = 94 \rightarrow m \approx 15.67°$. Exterior angle ≈ 113.7°. ∠X ≈ 47.3°, ∠Y = ∠Z ≈ 66.3°. Check: 47.3 + 66.3 + 66.3 ≈ 180 ✓.
Q13. Draw angle bisector from A to D. AB = AC (given), ∠BAD = ∠CAD (def. of bisector), AD = AD (reflexive) → △ABD ≅ △ACD (SAS) → ∠ABC = ∠ACB (CPCTC). ■
Q14. AB = AC (given), AD = AD (reflexive), ∠ADB = ∠ADC = 90° (AD ⊥ BC given) → △ADB ≅ △ADC (HL, or AAS) → BD = DC (CPCTC). ■
Q15. If a point lies on the bisector of an angle, then it is equidistant (perpendicular distance) from the two sides of the angle.
Q16. P lies on the angle bisector of ∠XYZ. Converse of the angle bisector theorem: if a point is equidistant from both sides, it lies on the bisector. PX = 7 (given perpendicular distance).
Q17. $2y + 3 = 4y - 9 \rightarrow 12 = 2y \rightarrow y = 6$. Perpendicular distance = 2(6) + 3 = 15.
Q18. P on perp. bisector → PA = PB (perp. bisector theorem). $3m + 1 = 5m - 9 \rightarrow 2m = 10 \rightarrow m = 5$. PA = 3(5) + 1 = 16.
Q19. Drop perpendiculars PE ⊥ AB (meeting at E) and PF ⊥ AC (meeting at F). ∠PEA = ∠PFA = 90° (def. of perp.). ∠EAP = ∠FAP (AD bisects ∠A → ∠BAD = ∠CAD, same angles). AP = AP (reflexive). △AEP ≅ △AFP (AAS). PE = PF (CPCTC). P is equidistant from AB and AC. ■
Q20. AB = AC → A is equidistant from B and C (equal sides) → A lies on the perpendicular bisector of BC (converse of perp. bisector theorem). A′B = A′C → same argument → A′ also lies on the perpendicular bisector of BC. Two distinct points (A and A′) both lying on the perpendicular bisector → line AA′ is the perpendicular bisector of BC (Postulate 1: two points determine a line). ■