

Check Your Understanding 2.1

Students begin this in class after notes. The problems are organized by the navigation instrument row for **Inductive Reasoning**. Students should attempt all three columns before the end of class.

Basic	Intermediate	Advanced
<i>Inductive Reasoning — Define and Use</i> 1. Define inductive reasoning in your own words. 2. Find the missing numbers in each pattern. Describe how each one works. 31, 27, 23, 19, ____, ____, ____ 2, 6, 5, 15, ____, 42, 41, ____	<i>Inductive Reasoning — nth Term and Explanation</i> 3. Write a rule to find the nth term of the sequence: 31, 27, 23, 19, ... 4. How many triangles are formed when you draw all possible diagonals from one vertex of a 35-gon? (Use the diagram as a guide and look for a pattern.) 	<i>Inductive Reasoning — Identify and Explain Flaws</i> 5. Mark Twain observed that the lower Mississippi River has been getting shorter over time — from over 1,200 miles in 1700 to 973 miles in 1875. He concluded that in 742 years, the river will be less than two miles long. a. What type of reasoning is Twain using? b. What is wrong with his argument? c. Does failing to find a counterexample to a conjecture prove that it is true? Explain.
<i>Spiral — Unit 1</i> 6. Find the distance between the points (3, 7) and (−1, −1).	<i>Spiral — Unit 1</i> 7. Simplify: $\frac{x^2y^{-2}}{x^5}$	<i>Extension — Construction and Existence</i> 8. In the handshake problem, we did not just find a pattern — we found a reason why the pattern must continue. Explain in your own words why understanding <i>why a pattern holds</i> is more powerful than only knowing that it holds.

CYU 2.1 — Answer Key

1. Definition: Inductive reasoning is the process of observing data, recognizing patterns, and making generalizations. A claim based on inductive reasoning is called a conjecture.

2a. 31, 27, 23, 19, 15, 11, 7 (subtract 4 each time)

2b. 2, 6, 5, 15, 14, 42, 41, 123 ($\times 3$, then -1 , alternating)

3. n th term: $31 - 4(n - 1)$ or equivalently $35 - 4n$

4. **33 triangles.** Drawing all diagonals from one vertex of an n -gon divides it into $n - 2$ triangles. For a 35-gon: $35 - 2 = 33$. Students can verify with small cases: 3-gon $\rightarrow$ 1 triangle, 4-gon $\rightarrow$ 2, 5-gon $\rightarrow$ 3, and so on.

5a. Inductive reasoning — he observed a pattern in a limited data set and extrapolated.

5b. He assumed the pattern was linear and would continue indefinitely. He used only two data points to establish the trend, which is insufficient. Rivers shorten through natural cutoffs but the process is not constant — the extrapolation is geographically absurd.

5c. No. Not finding a counterexample only means we haven't disproved the conjecture yet — it does not prove it. The circle task is the demonstration: five clean confirmations were not enough.

6. Distance = $\sqrt{80} = 4\sqrt{5} \approx 8.94$ using $d = \sqrt{[(3 - (-1))^2 + (7 - (-1))^2]} = \sqrt{[16 + 64]}$

7. $x^2y^{-2} / x^5 = y^{-2} / x^3 = 1 / (x^3y^2)$

8. Open response. Strong answers will explain: a pattern tells us *what* happens; an explanation tells us *why* it must happen. The explanation works for every case, not just the ones we checked. Bonus: some students may note that in the handshake problem, this connects to $1 + 2 + 3 + \dots + (n-1) = n(n-1)/2$.