

Unit 6

Similarity

Scaling, Shape, and Structure

8 days including assessment

Unit Navigation Instrument *Big Idea: Similarity is not a visual impression. It is a precise relationship — one that preserves angles, scales lengths proportionally, and enables reasoning across size.*

Subtopic	Basic	Intermediate	Advanced
Proportional Reasoning and Scale → <i>The engine behind similarity</i>	Use a scale factor or proportion to find unknown lengths when two figures are similar. Explain what stays the same and what changes under a dilation.	Set up and solve proportions involving similar figures. Recognize when a scale factor applies in both directions. Use both scale factor and proportion methods fluently.	Explain why both the scale factor method and the proportion method work, and articulate when each is more efficient. Identify errors in reasoning that stem from confusing the two.
Dilation and Similarity Defined → <i>From experience to definition</i>	State the definition of dilation and similarity. Identify the center, scale factor, and corresponding parts in a dilation.	Perform a dilation on a figure given a center and scale factor. Write a similarity statement with correct correspondence order. Explain what invariants a dilation preserves.	Prove that two figures are similar or explain why they are not. Identify a point of dilation from a diagram. Explain why a given transformation is or is not a dilation.
Triangle Similarity Shortcuts → <i>Proving similarity</i>	State the three triangle similarity shortcuts (AA, SAS~, SSS~). Identify which applies to a given pair of triangles.	Apply similarity shortcuts to find missing side lengths and angles. Explain why AA is sufficient while one or two angles alone is not. Explain the difference between SSS~ and SSS for congruence.	Justify that two triangles are similar using a similarity shortcut, then use that similarity to prove a further result. Explain why the inability to find a counterexample does not constitute proof.
Indirect Measurement	Use shadow or mirror similarity to	Apply indirect measurement	Justify why a given indirect

→ <i>Applying similarity</i>	set up a proportion and find an unknown height. State the assumptions required for each method.	methods to multi-step problems. Identify sources of error in real-world measurements and explain their geometric cause.	measurement method produces similar triangles, citing the specific similarity shortcut. Critique a flawed indirect measurement argument.
Scaling Laws → <i>similarity in different dimensions.</i>	State how perimeter, area, and volume scale under similarity given a scale factor k . Apply these relationships to find missing measurements.	Use scaling laws to work backward from area or volume ratios to find scale factors or linear dimensions. Explain why area and volume scale differently from length.	Explain why nature cannot simply scale organisms up, using the square-cube law. Apply scaling reasoning to a novel biological or physical scenario.

A standing expectation throughout this unit: before applying any property of similar triangles, students must first justify that the triangles are similar. Stating a ratio or setting up a proportion without first establishing similarity — by naming the shortcut and the conditions that satisfy it — is an incomplete argument. This expectation mirrors what was built in Units 3 and 4 and applies with the same force here. Push for it at the boards throughout the unit, put it in CYU problems, and make the justification step explicit in your own worked examples.

What This Unit Is About

Similarity is one of the central unifying ideas of the course. It connects proportional reasoning, transformation, triangle structure, measurement, and the mathematics of scaling into a single coherent thread. Students enter the unit with strong informal intuitions — from maps, photographs, shadows, and art — but without a precise framework for why those intuitions are reliable. The unit builds that framework from experience upward.

The unit moves in a deliberate arc. Lessons 6.1 and 6.2 establish similarity through physical experience and informal reasoning before any formal definition is introduced. Lesson 6.3 formalizes the triangle similarity shortcuts through counterexample investigation. Lesson 6.4 applies similarity to indirect measurement — two full days of work that give similarity real stakes. Lesson 6.5 closes the unit by examining how perimeter, area, and volume scale differently under similarity, connecting abstract geometry to biology, physics, and the structure of the natural world.

Equally important is the connection between similarity and proof. By the time students reach this unit, they have written flow chart and paragraph proofs in Units 3,4 and 5. That work applies here: establishing that two triangles are similar is the antecedent of every ratio and proportion calculation that follows. Students who set up proportions without first justifying similarity have skipped the essential logical step. This unit expects and reinforces that step throughout.

On Two Methods: Scale Factor and Proportion

From the first lesson, students will approach similarity problems in two ways: by computing a scale factor and multiplying, or by setting up a proportion and solving. Both methods are mathematically correct. Both will appear throughout the unit. Both must be understood.

In practice, scale factor tends to be the more intuitive approach — students gravitate toward it naturally, and it works well when the goal is to find a single missing length from a known ratio. But students who only know the scale factor method will struggle with certain problems: those where the scale factor is not immediately obvious, those involving multiple unknowns, and those in the indirect measurement lessons where the setup is more naturally expressed as a proportion. Students who rely exclusively on one method hit walls the other method handles cleanly.

Both methods should be addressed explicitly in consolidation during Lesson 6.2, not just mentioned. Show the same problem solved both ways. Ask students to identify which method they used and whether both produce the same answer. When a student uses one method, occasionally ask whether they could do it the other way. The goal is fluency with both, not preference for one.

Lesson Sequence Overview

Lesson 6.1: Similarity in Light and Shadow — Mount Rushmore proportional reasoning, flashlight shadow investigation, dilation from a point

Lesson 6.2: Dilation and Scale Factor — shadow without the flashlight, formal definitions, scale factor and proportion methods both established

Lesson 6.3: When Is Similarity Guaranteed? — counterexample task, AA/SAS~/SSS~ discovered, accepted as postulates

Lesson 6.4: Measuring the Inaccessible — Thales at the pyramid, outdoor investigation, mirror method (2 days)

Lesson 6.5: When Size Changes, What Changes With It? — scaling laws for perimeter, area, and volume, biological applications, Kleiber's Law

Lesson 6.6: Review and Assessment