Lesson 7.5 — Teacher Reference Notes

The Shape of Sine

Unit 7: Trigonometry • Day 8 • ≈45 minutes + self-evaluation • Closing lesson before Law of Sines

At a Glance	**Pacing:** Thinking task: plotting sin and cos (~25 min) • Consolidation: four representations (~10 min) • Desmos moment (~5 min) • Self-evaluation instrument (~10 min) • CYU **Materials:** Boards and markers • Class trig table (still posted) • Calculators (for values between the exact anchors) • Desmos animation ready to project • Navigation instruments distributed before self-evaluation **Nav instrument rows active:** Row 5 (Estimation) activates today — self-evaluation follows the Desmos moment **One task, not two:** Both y = sinθ and y = cosθ are plotted on the same axes simultaneously. The comparison between the two curves is the point — students who plot them separately miss the central observation. **Exact values only first:** Students plot only the values they can justify without a calculator first — 0°, 30°, 45°, 60°, 90° — then fill in from the class table. This keeps the exact values foregrounded. **Do not teach the wave:** The Desmos animation shows a full cycle (0° to 360° on the unit circle). Show it without narration, then ask the two questions. Leave students with an open question. The feeling that what they have learned opens into something larger is exactly right.

OBJECTIVES

By the end of this lesson students can:

- Plot sinθ and cosθ for 0° to 90° using exact values and class table values.
- Describe how sine increases and cosine decreases over this interval with a geometric explanation.
- Identify the crossing point at 45° and explain it geometrically.
- Name and connect all four representations of a trig function: verbal, symbolic, numerical, graphical.
- Use the graphs and exact anchors to evaluate the reasonableness of a trig value without a calculator.

> **Narrative Arc** — Open class — before releasing the task
>
> *"You have now seen sine and cosine as a definition in words, as a formula, and as a table of numbers. There is one representation you haven't seen yet. Today you build it."*

THINKING TASK — THE SHAPE OF SINE AND COSINE (≈25 MIN)

Task prompt — written on board

On the same set of axes, plot y = sinθ and y = cosθ for θ from 0° to 90°. Use only values you can justify without a calculator first — exact values from Lesson 7.4 and values from the class table. Label every point you plot with the angle. Describe in words what happens to each function as the angle increases. Where do the two curves cross? What is special about that angle? Explain geometrically, using a right triangle with hypotenuse 1, why sine and cosine must move in opposite directions.

⚠️ **What to watch for**

Minimum points: groups should use at minimum 0°, 30°, 45°, 60°, 90°. Encourage them to add values from the class table for the angles in between.

The crossing point: the curves cross at 45°. The geometric explanation is the key: at 45° the opposite and adjacent sides of a right triangle with hypotenuse 1 are equal, so sinθ = cosθ. For angles below 45° the adjacent is longer (cos > sin). For angles above 45° the opposite is longer (sin > cos).

Boundary values 0° and 90°: push groups to justify these from the triangle, not from memory. What happens to the opposite side as the angle approaches 0°? It shrinks to zero — so sin(0°) = 0. What happens to it as the angle approaches 90°? It approaches the full hypotenuse — so sin(90°) = 1. The triangle argument for the boundary values is more satisfying than looking them up.

Plotting order: ask groups to plot exact values first (0°, 30°, 45°, 60°, 90°), then fill in with class table values for any angles between. This keeps the distinction between exact and approximate visible.

CONSOLIDATION — FOUR REPRESENTATIONS (≈10 MIN)

Connect all four representations explicitly. Use this table as a reference during consolidation:

Representation	What it looks like for sine	What it reveals
Verbal	Sine is the ratio of the opposite side to the hypotenuse in a right triangle.	The geometric meaning — what the function is measuring.
Symbolic	sin θ = opp/hyp	The algebraic manipulations used in Lessons 7.2 and 7.3.
Numerical	The class trig table (Lesson 7.1)	Specific values at specific inputs. Reveals patterns (sine increasing, complement relationship).
Graphical	The curve y = sin θ plotted today	Rate of change — how the value is growing or slowing. The shape of the function across a continuous domain.

Each representation reveals something the others hide. The graph shows rate of change. The table shows specific values. The formula connects to geometry. The verbal definition connects to physical measurement.

The speech — say this during consolidation

'We have now seen sine and cosine in all four ways a function can be represented. Each one reveals something the others hide. The graph shows rate of change — you can see where sine is growing fast and where it slows. The table shows specific values. The formula connects to geometry. And the verbal definition keeps it rooted in what you actually measured in Lesson 7.1.'

THE DESMOS MOMENT (≈5 MIN)

How to run this

Project the Desmos animation: a point moving counterclockwise around a unit circle, with the sine wave tracing on a linked coordinate plane..

https://www.desmos.com/calculator/158125dc01

Then ask exactly two questions. Do not answer them.

The two questions — ask these, then leave them open

'Where does sine grow fastest? Where does it slow?'

'These waves have a name. You will study them formally in precalculus. What questions does this raise for you?'

Do not teach the wave. Do not explain period, amplitude, or phase shift. The point is not to introduce new content but to leave students with a felt sense that what they have learned opens into something larger. That feeling is mathematically correct and pedagogically important.

SELF-EVALUATION (≈10 MIN)

Distribute navigation instruments. Students complete a self-evaluation of all five rows. This is the primary purpose of the final block — not the CYU.

How to run the self-evaluation

Students work individually and in silence for 8 minutes. Ask them to mark each row honestly, then write one sentence answering: 'Which row do I feel least confident about, and what specifically is uncertain?'

Collect the instruments. Review before the next lesson. Any row where a significant number of students mark below Intermediate signals a need for targeted warm-up or additional practice before the assessment.

STUDENTS MAKE NOTES ON

Students make notes on:

- No need for notes today.

Narrative Arc — End of lesson — transition to Lesson 7.6 (Law of Sines)

"Right triangle trigonometry is complete. You can find any side or angle from any other combination, and you understand why. The next two lessons ask: what do you do when there is no right angle? The triangle might be acute or obtuse — no right angle guaranteed. You need new tools. The Law of Sines and Law of Cosines extend everything you have built to any triangle."

Connection Points	• Draws from Lesson 7.1: the class trig table (numerical representation) is one of the four representations named today • Draws from Lesson 7.4: the exact values at 0°, 30°, 45°, 60°, 90° are the required plotting points; the lesson only works if 7.4 is complete • Draws from Unit 2: the four representations of a function is a standard framework from algebra — naming it in this context connects geometry to students' prior algebraic experience • The Desmos wave foreshadows sinusoidal functions in precalculus — leave this open intentionally