

## Check Your Understanding 4.2

Show all work. Write CPCTC in full before using it. For proofs: flowchart or paragraph form only.

| Basic                                                                                                                                                                                                                                                              | Intermediate                                                                                                                                                                                                                                           | Advanced                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
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| <b>Row 1 Spiral — Triangle and Polygon Angle Relationships</b><br>→ Proving the laws                                                                                                                                                                               |                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>Basic — Spiral</i><br>1. In Lesson 4.1, the triangle angle sum proof was stated as a conditional. What was the condition — what assumption was required before the proof could be completed?<br>2. State the exterior angle theorem as a conditional statement. | <i>Intermediate — Spiral</i><br>3. An exterior angle of a triangle is $(6x + 4)^\circ$ and the two non-adjacent interior angles are $(2x + 15)^\circ$ and $(3x - 7)^\circ$ . Find $x$ and all four angle measures.                                     | <i>Advanced — Spiral</i><br>4. Explain why Euclid placed Proposition 31 (parallel construction) before Proposition 32 (triangle angle sum) in the Elements — even though the angle sum result was almost certainly discovered first. What does this tell us about the difference between discovery order and logical order?                                                                                                                                                                       |
| <b>Constructions</b><br>→ Building the tools                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>Basic</i><br>5. Describe the steps to copy an angle using compass and straightedge. You do not need to perform the construction.<br>6. Describe the steps to construct a line through point $P$ parallel to line $l$ .                                          | <i>Intermediate</i><br>7. Perform the angle copy construction. Draw your own angle below, label all points you create, and show all arc marks clearly.<br>8. Perform the parallel line construction. Draw your own line $l$ and point $P$ not on $l$ . | <i>Advanced</i><br>9. In the angle copy construction, identify the three pairs of equal sides that establish SSS. For each pair, state what makes them equal.<br>10. In the parallel line construction, which theorem from Unit 3 completes the proof? State it as a conditional. Which direction — theorem or converse — are you using?<br>11. Write the altitude proof in paragraph form. Explain what each step contributes and why the chain must go SSS → CPCTC → SAS → CPCTC → linear pair. |
| <b>Algebra Spiral</b><br>Solve equations — multi-step, variables on both sides                                                                                                                                                                                     | <i>Algebra Spiral</i><br>Solve each equation. Show your steps.<br>(a) $4x + 9 = 2x - 5$<br>(b) $3(2y + 1) = 5y + 12$<br>(c) $(3/4)n + 2 = (1/2)n - 6$                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

## Answers — CYU 4.2

### Row 1 Spiral

1. That a parallel line can be constructed through any point not on a given line (Playfair's axiom / Proposition 31).
2. If a side of a triangle is extended, then the exterior angle formed equals the sum of the two non-adjacent interior angles.
3.  $6x + 4 = (2x + 15) + (3x - 7) \rightarrow 6x + 4 = 5x + 8 \rightarrow x = 4$ . Exterior =  $28^\circ$ . Non-adjacent angles =  $23^\circ$  and  $5^\circ$ . Interior angle at vertex =  $180^\circ - 28^\circ = 152^\circ$ .
4. In a deductive system, every result must be justified by what came before it. Even if Euclid knew the angle sum result earlier, he could not place it in the Elements until the parallel construction on which it depends was established. Discovery order and logical order are independent.

### Constructions

5. Mark equal radii on both sides of the original angle. Transfer that radius to the new ray. Measure the chord across the original arc. Transfer that chord to the new arc. The intersection gives the copied angle.
6. Draw a transversal through P crossing l. At P, copy the angle the transversal makes with l (creating alternate interior angles). The new ray is parallel to l.
- 7–8. Construction problems — check that all arc marks are clearly drawn and all labeled points are present.
9. (1) Radii from the original vertex — equal by same compass opening. (2) Radii from the new vertex — same compass opening. (3) Chord lengths transferred — equal by same compass opening. SSS is established.
10. Alternate interior angles theorem (Unit 3). Converse direction: if alternate interior angles are equal, then the lines are parallel.
11. Drop perpendiculars from A to arc intersections P and Q.  $AP = AQ$  (equal radii, SSS).  $\angle PAH = \angle QAH$  (CPCTC).  $AH = AH$  (reflexive).  $\triangle APH \cong \triangle AQH$  (SAS).  $\angle AHP = \angle AHQ$  (CPCTC). Linear pair  $\rightarrow$  each =  $90^\circ$ . Therefore  $AH \perp PQ$ . ■

### Algebra Spiral

- (a)  $x = -7$
- (b)  $y = 9$
- (c)  $n = -32$