

Transversal Angle Relationships

Unit 2: Logic and Reasoning • Day 9 of 11

At a Glance

Pacing: ~40 min Tasks 1–3 + curated gallery walk • ~50 min consolidation (named relationships table + guided notes + CYU begins)

Materials: Boards and markers • Protractors (one per group for Task 1)

Nav instrument rows active: Rows 1–5 spiral • Row 6 (Transversal Angle Relationships) activates today

Critical instruction: Task 2 is a deduction task, not a measurement task. If students reach for a protractor to check logical conclusions, stop them gently but clearly.

OBJECTIVES

By the end of this lesson students can:

- Name and describe the four transversal angle relationships: corresponding, alternate interior, alternate exterior, co-interior.
- State each relationship as a conditional: ‘if two parallel lines are cut by a transversal, then...’
- Derive alternate interior and alternate exterior angle congruence from corresponding angles using vertical angles and linear pairs.
- Explain why the parallel assumption is not decoration — it is the premise that makes the deduction valid.

Narrative Arc — Before the lesson begins

“For four lessons we have been building logical tools in the abstract — conditionals, argument forms, valid reasoning. Today those tools meet geometry for the first time.”

“We are going to do something mathematicians have always done: first notice a pattern, then explain why it must be true. The observation comes from measurement. The explanation comes from logic. Watch for the moment when we cross from one to the other — that is the point of the lesson.”

TASKS

Task 1 — Conjecture the Relationships (Inductive Phase) (≈20 min)

Students draw two parallel lines in any orientation, draw a transversal intersecting both, label all 8 angles, measure with a protractor, and make at least three conjectures about which angles are congruent or supplementary.

 **During consolidation — show multiple orientations**

Show parallel lines diagonal, steep, and shallow. A relationship students found for horizontal parallel lines should still hold when the lines are rotated. If a student's conjecture does not hold after rotation, they have found something orientation-dependent rather than a genuine geometric property.

Task 2 — Deduction Follows Induction (Deductive Phase) (≈15 min)

Accept one result from Task 1 as a given: corresponding angles are congruent when lines are parallel. Using only vertical angles and linear pairs — tools from Unit 1 — construct a logical argument for:

- Why $\angle 7$ must be congruent to $\angle 2$ (alternate interior)
- Why $\angle 5$ must be congruent to $\angle 4$ (co-interior, via supplementary)

Discussion questions to pose during group work

- What type of statements are you using?
- What type of logical argument is this?
- What previous knowledge are you drawing on?
- Would the inverse of your conjectures be true?

⚠ What students are doing, named explicitly

Students are using conditional statements and the law of syllogism from Lesson 2.4. Name this during consolidation — they may not recognize it. The argument structure is the same one they practiced with Carroll puzzles. The tools transferred. This is the payoff of placing transversal relationships here rather than in Unit 1.

Contrast Task — What Happens Without Parallel Lines? (≈5 min)

Repeat the measurement task with two non-parallel lines cut by a transversal. Do the angle relationships still hold?

⚠ Do not skip this task

The relationships do not hold. This makes the conditional nature of the conjectures explicit: if the lines are parallel, then these relationships follow. If not, they do not. The 'if' is doing real work — and it is exactly what Postulate 5 establishes in Lesson 2.6. The parallel assumption is not decoration on the diagram; it is the premise that makes the deduction valid.

CONSOLIDATION — NAMED RELATIONSHIPS AND GUIDED NOTES

Consolidate the four named relationships. State each as a conditional. The conditional form is deliberate and will matter when students write geometric proofs in Unit 3.

Relationship	Position	Conditional form
Corresponding angles	Same position at each intersection — one at the top-left of each crossing.	<i>If two parallel lines are cut by a transversal, then corresponding angles are congruent.</i>
Alternate interior angles	Between the parallel lines, on opposite sides of the transversal.	<i>If two parallel lines are cut by a transversal, then alternate interior angles are congruent.</i>
Alternate exterior angles	Outside the parallel lines, on opposite sides of the transversal.	<i>If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.</i>
Co-interior angles (same-side interior)	Between the parallel lines, on the same side of the transversal.	<i>If two parallel lines are cut by a transversal, then co-interior angles are supplementary (sum to 180°).</i>

Name what happened in Task 2 explicitly

Students just proved geometric results deductively. They accepted one relationship as a premise and derived the others using tools they already had. That is a proof. It is informal in its current form — Unit 3 gives it more structure — but the intellectual act is the same. Say this directly.

Narrative Arc — End of Lesson 2.5 — before Lesson 2.6

“We just did something new. We took a result we found by measuring — a pattern we observed — and we derived other results from it using logic. We did not measure the other angles. We proved they had to be what they were, given what we accepted as a starting point.”

“But here is a question worth sitting with before next class: why did we get to accept corresponding angles as congruent in the first place? What made that a legitimate starting point? We observed it in many cases. But by now you know what that means: we have evidence, but not a proof. Next class we are going to build the system that makes our starting points legitimate.”

Students make notes on:

- All four transversal angle relationships: name, position description, and conditional form
- The deductive argument for at least one relationship — students record it in their own words
- Why the parallel assumption matters: the contrast task result

Connection Points

- Draws from Lesson 2.4: law of syllogism and conditional statements are the mechanism of the Task 2 deduction
- Draws from Unit 1: vertical angles and linear pairs are the only tools needed for the deduction
- Feeds into Lesson 2.6: the parallel assumption is a premise that needs legitimacy — Postulate 5 provides it
- Feeds into Unit 3: these relationships appear as justifications in proof from Lesson 3.3 onward