Check Your Understanding 4.3

Show all work. Write CPCTC in full before using it. For proofs: flowchart or paragraph form only.

Basic	Intermediate	Advanced
Row 1 Spiral — Triangle and Polygon Angle Relationships *→ Proving the laws*		
Basic — Spiral *1. An isosceles triangle has a base angle of 54°. Find all three angles.* *2. Find the exterior angle at one base vertex of the same triangle.*	*Intermediate — Spiral* *3. The apex angle of an isosceles triangle is (2x+ 10)°. The exterior angle at one base vertex is (4x-2)°. Find x and all three interior angles.*	*Advanced — Spiral* 4. An equilateral triangle has all three sides equal. Without assuming anything about the angles, use the base angles theorem to prove all three angles must be equal. Then find their measure and prove it using the triangle angle sum theorem.
Row 2 Spiral — Constructions *→ Building the tools*		
Basic — Spiral 5. What construction is used in the proof of the base angles theorem? In which lesson was it established?	*Intermediate — Spiral* 6. In isosceles triangle ABC with AB = AC, the angle bisector from A meets BC at D. Perform the angle bisector construction and label all points. Identify the triangle congruence that the construction guarantees.	*Advanced — Spiral* 7. In the altitude proof from Lesson 4.2, the chain ends with a linear pair argument giving 90°. In the base angles proof, the chain ends with SAS and CPCTC. Explain in one paragraph how these two chains differ in structure and what each one proves.
Base Angles Theorem *→ Proving and earning the converse*		
Basic *8. In triangle ABC, AB = AC. Name the two angles that must be equal.* *9. In triangle PQR, ∠P = ∠R. Name the two sides that must be equal.*	*Intermediate* *12. In isosceles triangle ABC with AB = AC, find the base angles if ∠BAC = 48°.* *13. In isosceles triangle XYZ with ∠X = ∠Z, find all three angles if ∠X = (3k + 5)° and ∠Z = (5k − 11)°.*	*Advanced* 15. Prove the base angles theorem. Your proof must state what is given and what is to be proven, name the construction used, cite every reason, and be written in flowchart or paragraph form.

Basic	Intermediate	Advanced
10. State the base angles theorem as a conditional statement. 11. State the converse of the base angles theorem as a conditional statement.	*14. The apex angle of an isosceles triangle is (2x + 10)°. Each base angle is (3x − 5)°. Find all three angles.*	16. The base angles theorem and its converse together form a biconditional. *(a) Write the biconditional statement.* *(b) Explain why a biconditional is stronger than either the theorem or its converse alone.* *(c) Give an example of a conditional from geometry whose converse is false.*
Algebra Spiral *Factor*	*Algebra Spiral* Factor each expression completely. *(a) $x^2 - 9x + 20$* *(b) $3m^2 + 15m$* *(c) $4n^2 - 36$*	

Answers — CYU 4.3

Row 1 Spiral

1. 54°, 54°, 72°.

2. 180° − 54° = 126°.

3. *4x−2 = (2x+10)+(base) and base=(180−(2x+10))/2=(170−2x)/2=85−x. Exterior=180−(85−x)=95+x. So 95+x=4x−2 → 97=3x → x=32.33.*

4. AB=AC → ∠B=∠C (base angles). AB=BC (equilateral) → ∠A=∠C. So ∠A=∠B=∠C. Triangle angle sum: 3∠A=180° → ∠A=60°.

Row 2 Spiral

5. Angle bisector construction. Established in Lesson 4.2 (Proposition 9).

6. Construction check — all arcs visible. △ABD ≅ △ACD by SAS.

7. Altitude proof: SSS gives triangle congruence → CPCTC gives equal angles → SAS gives second congruence → CPCTC gives supplementary equal angles → linear pair forces 90°. The final step uses angle measure reasoning. Base angles proof: SAS gives one triangle congruence → CPCTC gives the equal base angles directly. The final step is CPCTC alone — no linear pair needed.

Base Angles Theorem

8. ∠ABC and ∠ACB.

9. PQ and RQ.

10. If a triangle has two equal sides, then the angles opposite those sides are equal.

11. If a triangle has two equal angles, then the sides opposite those angles are equal.

12. 66°.

13. k = 8. ∠X = ∠Z = 29°. Third angle = 122°.

14. x = 22.5. Apex = 55°. Base angles = 62.5° each.

15. Given AB = AC. Draw angle bisector AD. AB = AC (given), ∠BAD = ∠CAD (def. bisector), AD = AD (reflexive). △ABD ≅ △ACD (SAS). ∠ABC = ∠ACB (CPCTC). ■

16a. A triangle has two equal sides if and only if it has two equal base angles.

16b. A biconditional allows reasoning in either direction without choosing which form applies. Both conditions are equivalent.

16c. Accept any valid example, e.g. 'If a triangle is equilateral, then it is isosceles.' Converse is false.

Algebra Spiral

(a) (x − 4)(x − 5)

(b) 3m(m + 5)

(c) 4(n − 3)(n + 3)