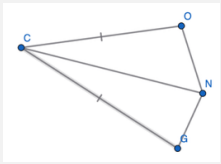
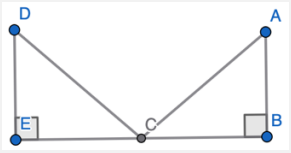
Check Your Understanding 3.5c

Complete each problem. Write CPCTC in full before using it. Flow chart or paragraph proof only.

Basic	Intermediate	Advanced
Row 3 Spiral — Triangle Congruence Shortcuts *→ The geometric facts we draw from to build arguments*		
Basic — Spiral For each set of given information, state whether it is sufficient to prove the triangles congruent. If yes, name the shortcut. If no, state exactly what is missing. *(a) △ABC and △DEF: ∠A = ∠D, ∠B = ∠E, BC = EF.* *(b) △PQR and △XYZ: PQ = XY, QR = YZ, ∠R = ∠Z.* *(c) △JKL and △MNO: ∠J = ∠M, JK = MN, ∠K = ∠N.*	*Intermediate — Spiral* In the diagram, ∠OCN and ∠GCN are formed by two triangles sharing vertex C. The given information is: OC = GC and NC bisects ∠OCG.  List all congruent parts (given, free, and derived). Write a congruence statement and state the shortcut.	*Advanced — Spiral* Given: BA ⊥ BE, DE ⊥ BE, C is the midpoint of BE, ∠DCB ≅ ∠ECA.  Identify all congruent parts and state the congruence shortcut that proves △ABC ≅ △DEC. Explain how ∠DCB and ∠ECA relate to the angles you actually need.
Row 4 Spiral — Reading a Diagram *→ Knowing what the diagram gives us for free*		
Basic — Spiral A diagram shows △OCN and △GCN sharing side CN. The following marks are shown: OC = GC and the angle bisector from C. List what is given, what is free, and what can be derived. Which shortcut applies?	*Intermediate — Spiral* For the diagram in the advanced Row 3 problem (BA ⊥ BE, DE ⊥ BE), a student claims ∠ABC = ∠DEC = 90° as free information. Is this correct? Explain, and state what makes it true.	*Advanced — Spiral* ∠DCB and ∠ECA are given as congruent. The angles you need for the proof are ∠ABC and ∠DEC (the right angles) and ∠ACB and ∠DCE (the angles at C inside each triangle). *(a) Explain how ∠DCB and ∠ECA are related to ∠ACB and ∠DCE using the Angle Addition Postulate.* *(b) Why does this mean ∠ACB = ∠DCE even though they are not the angles you were given?*

Basic	Intermediate	Advanced

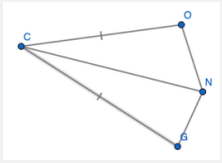
Row 5 Spiral — Writing Proofs

→ *Constructing arguments that cannot reasonably be rejected*

Basic — Scaffolded

Given: OC = GC, NC bisects ∠OCG.

Prove: ∠OCN ≅ ∠GCN.



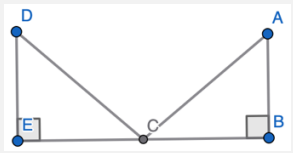
Fill in the missing reasons.

Statement	Reason
OC = GC	_____
∠OCN = ∠GCN	_____
CN = CN	_____
△OCN ≅ △GCN	_____
∠OCN ≅ ∠GCN	_____

Intermediate — Complete Proof

Given: BA ⊥ BE, DE ⊥ BE, C is the midpoint of BE, ∠DCB ≅ ∠ECA.

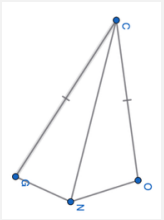
Prove: △ABC ≅ △DEC.



Advanced — No Intermediate Steps Given

Given: OC = GC, NC bisects ∠OCG.

Prove: ON = GN.



Identify the intermediate steps yourself. Show your planning, then write the complete proof.

Row 6 Spiral — Planning Proofs

→ *Seeing the path before writing the argument*

Basic — Spiral

You want to prove ON = GN using the diagram from Row 5.

(a) What is the final conclusion?

(b) What result would give you that conclusion?

(c) What intermediate result do you need before that?

Intermediate — Spiral

Plan the proof of ON = GN using backward reasoning. Work from the conclusion back to the given. Write each step of the plan.

Advanced — Spiral

Given: BA ⊥ BE, DE ⊥ BE, C is the midpoint of BE, ∠DCB ≅ ∠ECA.

Before writing the proof of △ABC ≅ △DEC, show your complete plan. Identify every intermediate step, every piece of given and free information you use, and the final shortcut.

Algebra Spiral

Solve factorable quadratics

Algebra Spiral

Solve each quadratic equation by factoring.

(a) $x^2 + 5x + 6 = 0$

(b) $2x^2 - 7x - 15 = 0$

(c) $3x^2 = 12x$

CYU 3.5c — Answer Key (Teacher Copy)

Row 3 Spiral

(a) AAS — sufficient.

(b) SSA — not sufficient.

(c) ASA — sufficient.

Intermediate: Given: OC = GC, ∠OCN = ∠GCN (angle bisector). Free: CN = CN (reflexive). Shortcut: SAS. △OCN ≅ △GCN.

Advanced: Free: ∠ABC = ∠DEC = 90° (perpendicularity given). Free: BC = CE (midpoint). Convert: ∠DCB = ∠ECA (given) → ∠ACB = ∠DCE (supplement/complement reasoning — see Row 4 Advanced). ASA: △ABC ≅ △DEC.

Row 4 Spiral

Basic: Given: OC = GC, ∠OCN = ∠GCN (bisector). Free: CN = CN (reflexive). SAS.

Intermediate: Not free — it follows from the given BA ⊥ BE and DE ⊥ BE, which is stated in the problem. The right angles are derived from the given, not free from the diagram.

Adv (a): ∠DCB + ∠ACB = ∠DCA (angle addition). ∠ECA + ∠DCE = ∠DCA. Since ∠DCB = ∠ECA (given) and both form the same total angle, ∠ACB = ∠DCE.

Adv (b): Equal quantities subtracted from equal totals give equal results (Common Notion 3).

Row 5 — Writing Proofs

Basic reasons: Given; Definition of angle bisector; Reflexive property; SAS; CPCTC (Corresponding Parts of Congruent Triangles are Congruent).

Intermediate: ∠ABC = ∠DEC = 90° (perpendicular lines). BC = CE (midpoint). ∠DCB = ∠ECA (given) → ∠ACB = ∠DCE (angle addition + Common Notion 3). △ABC ≅ △DEC by AAS. ■

Advanced: Prove △OCN ≅ △GCN (SAS) → CPCTC → ON = GN. ■

Row 6 — Planning Proofs

Basic: (a) ON = GN. (b) CPCTC from △OCN ≅ △GCN. (c) SAS: OC = GC, ∠OCN = ∠GCN, CN = CN.

Intermediate: Want ON = GN → CPCTC → △OCN ≅ △GCN → SAS → OC = GC (given), ∠OCN = ∠GCN (bisector), CN = CN (reflexive) → all at given. ■

Advanced: Intermediate steps: (1) ∠ABC = ∠DEC = 90° from perpendicularity. (2) BC = CE from midpoint. (3) ∠DCB = ∠ECA given → ∠ACB = ∠DCE by angle addition. (4) AAS: ∠ABC = ∠DEC, ∠ACB = ∠DCE, BC = CE. ■

Algebra Spiral

(a) $x = -2$ or $x = -3$

(b) $x = 5$ or $x = -3/2$

(c) $x = 0$ or $x = 4$