Unit 1 — Unit Review

Foundations, Coordinates, and the First Acts of Proof

Name: ______________________________

Period: ________ Date: ____________

Show all work. Use correct notation throughout.

What Makes a Good Definition *→ Classification and differentiation*	**Basic**	**Intermediate**	**Advanced**

1 *Basic*

A student defines a ray as: ‘Part of a line.’

(a) What does this definition classify correctly?

(b) What does it fail to differentiate from? Name at least one other geometric object that also fits this definition.

2 *Intermediate*

Write a definition for ‘acute triangle’ using classification and differentiation.

(a) Test your definition: does it include all acute triangles?

(b) Does it correctly exclude right triangles and obtuse triangles?

(c) Give one example and one non-example.

3 *Advanced*

Invent a new geometric object. Give it a name. Write a formal definition using classification and differentiation.

Then provide two examples and two non-examples that test the boundaries of your definition. At least one non-example should be a near-miss — something that almost fits but doesn’t.

The Foundational Objects *→ Naming what we reason about*	**Basic**	**Intermediate**	**Advanced**

4 *Basic*

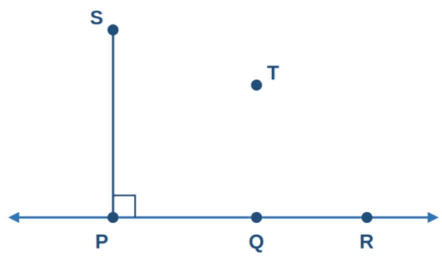
Define each term in your own words and draw a small diagram for each using correct notation.

(a) Parallel lines

(b) Perpendicular lines

(c) Collinear points

5 *Intermediate*



Using the diagram, name each of the following with correct notation:
(a) Two different rays that share endpoint P.
(b) A pair of perpendicular segments.
(c) A set of three collinear points.
(d) A pair of non-collinear points (with respect to P, Q, R).

6 *Advanced*

Why must every mathematical system include some undefined terms?
(a) Use the classification-and-differentiation framework from Lesson 1.1 to explain why the chain of definitions must eventually stop.
(b) Give a specific example of what would go wrong if we tried to define ‘point.’

Distance and Midpoint in the Plane *→ Measuring what we reason about*	**Basic**	**Intermediate**	**Advanced**

7 *Basic*

(a) Find the distance between (−4, 1) and (2, −7).
(b) Find the midpoint of the segment from (−4, 1) to (2, −7).

8 *Intermediate*

(a) Segment RS has one endpoint at R(−1, 5) and midpoint at M(3, 2). Find S and the length of segment RS.
(b) A student says the distance formula is ‘basically just the Pythagorean theorem on a coordinate plane.’ Explain why this is correct. Draw a diagram that shows the right triangle hiding inside a distance calculation.

9 *Advanced*

Points A(1, 1) and B(9, 7) are the endpoints of a diameter of a circle.
(a) Find the center and radius of the circle.
(b) Determine whether point C(5, 7) lies on, inside, or outside the circle. Show your work.
(c) In one sentence: what geometric tool are you using, and why does it work here?

Polygons and Area in the Plane *→ Applying measurement to figures*	Basic	Intermediate	Advanced

10 *Basic*

(a) Classify this polygon by number of sides and concavity.
(b) A triangle has vertices A(0, 0), B(8, 0), C(3, 7). Find its area.

11 *Intermediate*

A trapezoid has vertices A(0, 0), B(10, 0), C(8, 4), D(2, 4).
(a) Find the area by decomposing it into simpler shapes. Show your work.
(b) Find the perimeter. Show your distance formula work for each slanted side.

12 *Advanced*

(a) Explain why the formula Area = ½ × base × height for a triangle can be derived from the area formula for a rectangle. Draw a diagram that shows the derivation clearly.
(b) A triangle has vertices at (0, 0), (5, 0), and (1, 6). Find the area using two different methods and confirm they agree.

Angle Relationships *→ The first deductive results*	Basic	Intermediate	Advanced

13 *Basic*

(a) ∠A and ∠B are complementary. m∠A = 37°. Find m∠B.
(b) ∠C and ∠D form a linear pair. m∠C = (4x + 6)° and m∠D = (2x + 24)°. Find x and both angle measures.

14 *Intermediate*

(a) Two lines intersect. One of the four angles formed measures (7x − 10)°. The angle adjacent to it measures (3x + 40)°. Find x and all four angle measures.
(b) ∠1 and ∠2 are supplementary. ∠2 and ∠3 are also supplementary. What can you conclude about ∠1 and ∠3? Justify your answer with a complete argument.

15 *Advanced*

Three lines meet at a single point, forming six angles. One of the angles measures 50° and an adjacent angle measures 70°.

(a) Find all six angle measures. Draw a diagram.
(b) In any arrangement where three lines meet at a point, how many angle measurements do you need to know in order to determine all six? Explain why.

Algebra Spiral *Equations, exponents, and combining like terms*	**Basic**	**Intermediate**	**Advanced**

16 *Basic*
(a) Solve: $4(2x - 3) - 5x = 3x + 6$
(b) Simplify: $(6a^2b^3)^2 \div (9a^3b)$
(c) Solve: $\frac{2}{3}x - 5 = \frac{1}{4}x + 3$

Answers — Unit 1 Review

Row 1 — What Makes a Good Definition

1. (a) Classifies correctly — a ray is part of a line. (b) Fails to differentiate from a line segment, which is also part of a line. The definition does not mention the key feature: a ray has one endpoint and extends infinitely in one direction.

2. An acute triangle is a triangle (classification) in which all three angles are acute — that is, all three angles measure less than 90° (differentiation). Test: includes all acute triangles (all angles < 90°). Excludes right triangles (one angle = 90°) and obtuse triangles (one angle > 90°). Example: a triangle with angles 50°, 60°, 70°. Non-example: a triangle with angles 30°, 60°, 90°.

3. Accept any creative answer with a proper classification/differentiation structure, two valid examples, and two valid non-examples. Award full credit for a near-miss non-example that genuinely tests the boundary of the definition.

Row 2 — The Foundational Objects

4. Parallel: two lines in the same plane that never intersect (∥ notation, diagram with arrows). Perpendicular: two lines that intersect at 90° (⊥ notation, diagram with square corner mark). Collinear: points that lie on the same line (diagram with 3+ points on one line).

5. Answers depend on diagram. Example: (a) Ray PQ and ray PS. (b) Segment PS ⊥ segment PQ (or PR). (c) P, Q, R. (d) S (or T) with respect to the line PQR.

6. (a) Every definition classifies a term using a larger category. That category term must also be defined, requiring yet another category. This chain cannot continue forever without becoming circular. Undefined terms stop the chain. (b) To define 'point,' you might say 'a location in space' — but then 'location' and 'space' need definitions. 'Location' might be defined as 'a point in space,' which is circular.

Row 3 — Distance and Midpoint

7. (a) $d = \sqrt{(2-(-4))^2 + (-7-1)^2} = \sqrt{36+64} = \sqrt{100} = 10$. (b) $M = ((-4+2)/2, (1+(-7))/2) = (-1, -3)$.

8. (a) $M = ((-1+x)/2, (5+y)/2) = (3, 2)$. So $x = 7$, $y = -1$. $S = (7, -1)$. $RS = \sqrt{(7-(-1))^2 + (-1-5)^2} = \sqrt{64+36} = 10$. Full segment = 10. (b) The distance formula computes $\sqrt{(\Delta x)^2+(\Delta y)^2}$. Draw a right triangle with horizontal leg Δx and vertical leg Δy — the distance is the hypotenuse. This is exactly $a^2 + b^2 = c^2$.

9. (a) Center = midpoint of AB = $((1+9)/2, (1+7)/2) = (5, 4)$. Radius = half the diameter = $\frac{1}{2}\sqrt{(9-1)^2+(7-1)^2} = \frac{1}{2}\sqrt{64+36} = \frac{1}{2}(10) = 5$. (b) Distance from (5,4) to (5,7) = $\sqrt{0+9} = 3$. Since $3 < 5$ (radius), point C is inside the circle. (c) The distance formula — comparing a point's distance from the center to the radius determines whether the point is on, inside, or outside the circle.

Row 4 — Polygons and Area

10. (a) Concave hexagon. (b) Base = 8, height = 7. Area = $\frac{1}{2} \times 8 \times 7 = 28$.

11. (a) Decompose: rectangle 0–6 base × 4 height minus two corner triangles. Or: two right triangles on the ends plus a rectangle in the middle. Trapezoid area = $\frac{1}{2}(10+6)(4) = 32$. (b) AB = 10, CD = 6. AD: $\sqrt{(2-0)^2+(4-0)^2} = \sqrt{20} = 2\sqrt{5}$. BC: $\sqrt{(8-10)^2+(4-0)^2} = \sqrt{20} = 2\sqrt{5}$. Perimeter = $10 + 6 + 2(2\sqrt{5}) = 16 + 4\sqrt{5} \approx 24.94$.

12. (a) Draw a rectangle around the triangle with the base as one side. The triangle fills exactly half the rectangle (the other half is cut away by the hypotenuse). So Area = ½ × base × height. Diagram: rectangle with diagonal = hypotenuse of the triangle, two right triangles on either side. (b) Method 1: base = 5, height = 6. Area = $\frac{1}{2}(5)(6) = 15$. Method 2: bounding rectangle 5×6 = 30. Left triangle: $\frac{1}{2}(1)(6) = 3$. Right triangle: $\frac{1}{2}(4)(6) = 12$. Area = $30 - 3 - 12 = 15$. ✓

Row 5 — Angle Relationships

13. (a) $m\angle B = 90° - 37° = 53°$. (b) $4x+6+2x+24 = 180$. $6x+30 = 180$. $x = 25$. $m\angle C = 106°$, $m\angle D = 74°$.

14. (a) Adjacent angles are supplementary (linear pair): $(7x-10) + (3x+40) = 180$. $10x+30 = 180$. $x = 15$. The four angles: 95°, 85°, 95°, 85°. (b) $\angle 1 + \angle 2 = 180°$ and $\angle 2 + \angle 3 = 180°$. Both equal 180°, so $\angle 1 + \angle 2 = \angle 2 + \angle 3$. Subtract $\angle 2$: $\angle 1 = \angle 3$. Supplements of the same angle are congruent.

15. (a) Label the six angles around the point. The 50° angle and 70° angle are adjacent, so the third angle in that half is $180° - 50° - 70° = 60°$. The vertical angles give the other three: 50°, 70°, 60°. The six angles are: 50°, 70°, 60°, 50°, 70°, 60°. (b) You need exactly two measurements (two non-adjacent angles, or two adjacent angles). The third angle on each side is determined by the straight-line constraint (angles on a straight line sum to 180°), and vertical angles give the other three for free.

Algebra Spiral

16. (a) $8x - 12 - 5x = 3x + 6$. $3x - 12 = 3x + 6$. $-12 = 6$. No solution (contradiction). (b) $36a^4b^6 \div 9a^3b = 4ab^5$. (c) Multiply by 12: $8x - 60 = 3x + 36$. $5x = 96$. $x = 96/5 = 19.2$.