Lesson 3.5b — Teacher Reference Notes

Proof Presentations

Unit 3: Proof and Congruence • Day 6 of 9

At a Glance	
	Pacing: Phase 1: group proof writing (~35 min) • Phase 2: presentations and peer critique (~30 min) • Individual follow-up (~20 min) • Collect work (5 min) **Materials:** Boards and markers • Proof assignment cards printed and cut (one per group — assign before class) • Navigation instruments available to students throughout **Nav instrument rows active:** Rows 1–5 spiral • Row 6 (Writing Complete Proofs) is fully active today **Assign problems before class:** Use the assignment guide below. Read the teacher notes for Problems 5 and 8 before assigning. **Groups:** Visibly random throughout.

OBJECTIVES

By the end of this lesson students can:

- Write a complete flow chart or paragraph proof independently from a cold problem with no scaffolding.
- Present a proof to peers and respond to questions about specific justifications.
- Critique a peer proof by asking questions about steps that appear unjustified or unclear.
- Recognize when a proof cannot be completed with the theorems currently available — and articulate why.

PROBLEM ASSIGNMENT GUIDE

#	Given / Prove	Difficulty	Teacher Notes
1	Given: CO≅CG and ON≅NG Prove: ∠OCN≅∠GCN	Accessible	SSS then CPCTC. Good first assignment. Clean argument with no tricky steps.
2	Given: RN≅NE and ∠R≅∠E Prove: RG≅AE	Accessible	AAS then CPCTC. Clean argument.
3	Given: T is midpoint of HI, WI∥HC Prove: T is midpoint of WC	Moderate	Requires CPCTC then midpoint definition. Two-step chain after congruence.
4	Given: GK∥ML and GK≅ML Prove: ∠G≅∠L	Moderate	Alternate interior angles + SAS then CPCTC.
5	Given: GL bisects KM and ∠KGM Prove: KG≅MG	★ See note	CANNOT be completed with current tools. See teacher note below.
6	Given: AB∥DC and ∠A≅∠C Prove: AB≅DC	Moderate	AAS then CPCTC. Alternate interior angles from parallel lines needed.

7	Given: OG bisects ∠EOM and EO≅MO Prove: ∠M≅∠E	Moderate	SAS then CPCTC. Bisector gives the included angle.
8	Given: BA⊥BE, DE⊥BE, C midpoint of BE, ∠DCB≅∠ECA Prove: △ABC≅△DEC	★★ Hardest	Angle subtraction via Common Notions. Assign to strongest group only. See teacher note below.

TEACHER NOTES — CRITICAL PROBLEMS

★ Problem 5 — The Proof That Cannot Be Completed Yet

Problem 5 asks students to prove KG ≅ MG given that GL bisects KM and ∠KGM. This proof cannot be completed with the theorems currently in the system.

To prove KG ≅ MG using CPCTC, students need two congruent triangles. The natural approach is SAS or ASA. But either approach requires knowing that the angles at the bisection point are right angles — that GL is perpendicular to KM. That is true, but it has not been proven in our system. It requires an intermediate theorem not yet established.

One student during Year 1 conjectured: 'If it bisects the angle and the side, then it must be perpendicular.' This conjecture is correct. But it is an unproven proposition — and using it as if it were established is precisely the error we are training students to avoid.

The mathematical principle at stake

This connects directly to the structure of Euclid's Elements: propositions must be proven in order. We cannot use Proposition 7 to prove Proposition 4, even if Proposition 7 turns out to be true. The same principle governs all of mathematics. Many results have been proven to follow from the Riemann Hypothesis. They cannot be accepted until the Hypothesis itself is established — because they hang on the truth value of an unproven antecedent.

Teacher decision: you may leave this problem out of the assignment, or assign it deliberately to a group you trust to engage seriously with the conversation it will generate.

If a group discovers they cannot complete the proof, do not rescue them. Ask: 'What would you need to know to get there? Do we have that yet?' If time allows, ask the class what theorem they would need to prove first. This is a preview of Unit 4.

★★ Problem 8 — The Hardest Problem (assign to strongest group only)

The key move: students are given ∠DCB ≅ ∠ECA. These are not the angles they need.

∠DCB = ∠DCA + ∠ACB = ∠2 + ∠3
∠ECA = ∠ECB + ∠BCA = ∠1 + ∠2

Since the wholes are equal and ∠2 is common to both, Common Notion 3 (if equals are subtracted from equals, the remainders are equal) gives ∠1 ≅ ∠3 — that is, ∠ACB ≅ ∠DCE.

Once students have ∠1 ≅ ∠3, the rest assembles: the right angles at B and E (from BA ⊥ BE and DE ⊥ BE), and BC = CE (from C being the midpoint of BE), give ASA.

Prompt if stuck

'You have ∠DCB and ∠ECA given as equal. Can you express each of those angles as the sum of two smaller angles? What do they share?'

PHASE 1 — GROUP PROOF WRITING (≈35 MIN)

Distribute proof cards. Each group writes their assigned proof on the board as a flow chart or paragraph proof.

Say to the class:

Instructions to read aloud

"Your group has been assigned a proof. Write it on the board — flow chart or paragraph, your choice. Every statement needs a justification. Every claim needs a source. When you are ready, you will present your proof to the class. Be prepared to explain every step and answer questions."

⚠️ Resisting the urge to answer

Students will ask for help. The instinct to answer directly removes the cognitive work the lesson is designed to produce.

When a group asks a question, redirect with a question of your own:

- *"What do you know from the given? What does that allow you to say?"*
- *"What would have to be true before you could write that step?"*
- *"Is there an intermediate step you might need to prove first?"*

A group that is genuinely stuck after sustained effort needs a hint, not an answer. The hint should be the smallest possible push: 'Have you thought about what the diagram gives you for free?'

Circulate to all groups. Note which groups have clean justified arguments and which have steps that rest on assumptions. You will use this during presentations.

PHASE 2 — PRESENTATIONS AND PEER CRITIQUE (≈30 MIN)

Groups present one at a time. The presenting group walks the class through their argument step by step. The class listens for unjustified steps and asks questions.

Say to the class:

Instructions to read aloud

"As each group presents, your job is to listen carefully. If a step is stated without a clear justification — or if the justification doesn't actually support the claim — ask about it. Ask with a question, not a correction: 'How do you know that?' or 'What tells you those angles are congruent?'"

If Problem 5 is presented

If the group assigned Problem 5 presents and could not complete the proof, do not treat this as failure. This is one of the richest moments in the unit.

Ask the class: 'This group could not complete this proof. Why not? Is there something wrong with their reasoning, or is there something wrong with the problem?' Let the conversation develop.

Guide students toward: this proof is incomplete because we are missing an intermediate theorem. We would need to first prove that a segment bisecting both a side and the angle at the vertex must be perpendicular to that side. We cannot borrow from a result we have not yet proven, even if we believe it is true.

INDIVIDUAL FOLLOW-UP (≈20 MIN)

After presentations, students individually write a proof for a problem that a different group presented. Students choose which problem to attempt based on their navigation instrument self-assessment.

Say to the class:

Instructions to read aloud

"Choose one proof from today's presentations — not the one your group was assigned. Write a complete proof on your own: flow chart or paragraph. Use your navigation instrument to choose your level. If you are aiming for Intermediate on Row 6, choose a moderate problem. If you are aiming for Advanced, choose one of the harder ones."

What to watch for during individual follow-up

The group writing phase develops the proof with collective support. The individual follow-up is where students find out what they actually understand independently.

Common errors from Year 1 to watch for:

- *Correctly identifies the shortcut but does not specify which parts are congruent and how each is established.*
- *Uses alternate interior angles without citing a given of parallel lines.*
- *Names angles incorrectly — correspondence order errors.*

Collect this work at the end of class. Before Lesson 3.5c, scan for the most common errors and plan a brief warm-up that addresses them without singling out individuals.

STUDENTS MAKE NOTES ON

Students make notes on:

- No new notes are made in Lesson 3.5b. Students work from the notes made in 3.5a throughout.
- Individual follow-up proofs are collected — these serve as the teacher's diagnostic data for Lesson 3.5c.

Connection Points	• Draws from Lesson 3.5a: all three proof planning strategies are in play; students apply them independently for the first time • Draws from Lessons 3.2–3.3: shortcuts and CPCTC are the tools used in every proof • Problem 5 draws from Unit 2: unproven propositions cannot be used as premises, regardless of whether we believe them to be true • Feeds into Lesson 3.5c: individual follow-up work is the diagnostic data for differentiating scaffolded practice