

What Is a Definition?

Unit 1: Foundations of Geometry • Day 1 of 2

At a Glance

Pacing: ~45 min shoe task + debate • ~45 min consolidation (guided notes, circular definition moment, undefined terms)

Materials: Shoe collection (see task note) • Boards and markers • Guided note sheet (distribute at consolidation)

Nav instrument rows active: Row 1 (What makes a good definition) and Row 2 (The foundational objects) activate today — aim to complete all vocabulary by end of Day 1

Note on guided notes: This consolidation uses a guided note sheet — word and picture provided, students fill in definitions. Be honest with students: today we are recording definitions carefully because we need them. Consolidation will look different later in the course.

OBJECTIVES

By the end of this lesson students:

- Can explain what makes a good definition: classification and differentiation.
- Can explain why circular definitions arise and why undefined terms are necessary.
- Have definitions for most or all foundational objects (point, line, ray, segment, plane, collinear, coplanar, parallel, perpendicular).

TASK

The Shoe Task (adapted from Math Medic, mathmedic.com) (≈40 min)

Preparation

Bring a collection of footwear to class. Include at least: one clear shoe, one clear non-shoe, and several items in the ambiguous middle — sandal, slipper, thick sock, doll shoe, ankle bracelet, rubber boot, ballet slipper. The ambiguous items are where the task lives.

Say: 'In groups: write a definition of a shoe.'

Once groups have definitions: present each object one at a time. Each group decides whether their definition includes or excludes it and must justify their answer. After testing, give groups time to revise. Then attempt to build one class definition.

What the debate is doing

Students will argue. They will poke holes in each other's language. They will revise. This is not a diversion from the lesson — it is the lesson. The arguments students make about

shoes are structurally identical to the arguments they will make about geometric objects for the rest of the year.

Consolidation question after the task: what made a definition work or fail? What two elements did the best definitions have?

Target answer: Classification (what group does it belong to?) + Differentiation (what makes it distinct within that group?). The Dictionary.com shoe definition is a good model: 'an external covering for the human foot [classification], usually of leather, consisting of a more or less stiff sole and a higher part ending around the ankle [differentiation].'

CONSOLIDATION — THE CIRCULAR DEFINITION MOMENT

Move from the shoe task directly to the foundational definitions. Begin with the circular definition demonstration.

⚠ The circular definition moment — how to run it

Take a key word from one group's definition and ask another student to define it. Take a word from their definition and ask someone else. Continue until the chain loops back to an earlier word. One student says their word means 'to create.' You ask what 'create' means. 'To make.' What does 'make' mean? 'Well... to create.'

If it does not arise naturally, construct it deliberately using 'line' as the starting word. The experience of arriving at the loop is what makes it stick. Being told that loops exist does not.

The philosophical payoff — worth stating directly

Every system of definitions must start somewhere. If you define every word, you eventually loop or regress infinitely. The only way out is to choose a small number of terms so basic that we accept them and build everything else from there.

Euclid attempted to define his foundational terms. His definition of a point was 'that which has no part.' His definition of a line was 'a length without breadth.' These are descriptions, not real definitions — they tell us almost nothing while creating the appearance of having said something. Modern geometry recognized this honestly: point, line, and plane are declared undefined terms. We will do the same.

After the circular definition moment, distribute the guided note sheet and work through the foundational vocabulary together. Priority for today: point, line, ray, segment, plane. Complete collinear, coplanar, parallel, perpendicular if time allows. Any remaining definitions carry into Day 2 briefly before the card task.

DEFINITIONS REFERENCE — TEACHER COPY

Undefined terms: accepted without definition. Students should understand why these cannot be defined, not just memorize the label.

Undefined terms

Point: A location in space with no size, dimension, or extent. Named with a capital letter.

Line: A straight, continuous arrangement of infinitely many points extending in both directions without end. Named with any two points on it (with double-headed arrow above) or a lowercase letter.

Plane: A flat surface extending infinitely in all directions, with no thickness. Determined by any three non-collinear points. Named with a capital script letter or three non-collinear points on it.

Defined terms: built from the undefined terms and from each other.

Defined terms

Line segment (segment): The part of a line consisting of two endpoints and all points between them. Named by its two endpoints (with a bar above, no arrows).

Ray: A part of a line that begins at an endpoint and extends infinitely in one direction. Named by its endpoint first, then any other point on it (with a single arrow above pointing right).

Collinear points: Points that lie on the same line.

Coplanar points: Points that lie in the same plane.

Parallel lines: Lines in the same plane that never intersect. Symbol: $\parallel$

Perpendicular lines: Lines that intersect to form four right angles (90° angles). Symbol: $\perp$

Congruent: Having exactly the same shape and size. For segments: equal length. Symbol: $\cong$ (introduced briefly here; unpacked fully in Unit 3.)

Notation quick reference

$\overleftrightarrow{AB}$ (line): double-headed arrow above AB

$\overrightarrow{AB}$ (ray): single arrow above AB, endpoint listed first

$\overline{AB}$ (segment): bar above AB, no arrows

AB (length/distance): no symbol above — just the two letters; represents a number

$m\angle ABC$ (angle measure): m before the angle symbol; represents a number in degrees

Three-legged chair — use this for the plane definition

Ask students how many legs a chair should have if they want it to never rock. Three. Three legs always meet a flat surface stably — they determine exactly one plane. Four legs might not all touch if the floor is slightly uneven, because four points do not necessarily lie in the same plane. Two points define a line, which can lie in infinitely many planes. Three non-collinear points pin the plane to a specific orientation.

 Narrative Arc — End of Day 1 — after the circular definition moment

“We just watched something interesting happen. We tried to define a word, and to do that we needed another word, and to define that word we needed the first one back. We went in a circle.”

“This is not a problem we can solve by trying harder. It is a fundamental constraint on any system of definitions. At some point, you have to stop defining and just agree on some starting terms — terms so basic that trying to define them only makes things worse. Euclid tried. His definitions were so spare they functioned more as descriptions than real definitions. Modern mathematicians eventually recognized this honestly and declared these terms undefined: accepted without definition, used as building blocks for everything else. We are going to do the same thing.”

 Students make notes on:

- Two elements of a good definition: classification and differentiation
- Why circular definitions arise and why undefined terms are necessary
- Definitions of foundational objects: point, line, ray, segment, plane — with pictures and notation
- Collinear, coplanar, parallel, perpendicular if time allows (complete in Day 2 if needed)

Connection Points

- Draws from Unit 0: the shoe task is classification and differentiation applied to an everyday object — the same logical structure used in the bucket sort
- Feeds into Day 2: notation is built on the vocabulary established today
- Feeds into Unit 3: Euclid’s undefined terms return when we study his construction proofs