Lesson 3.2 — Teacher Reference Notes

What Makes Triangles the Same?

Unit 3: Proof and Congruence • Day 2 of 9

At a Glance	
	Pacing: Task 1 (~10 min) • Task 2 (~35 min) • Curated gallery walk (~8 min) • Consolidation (~25 min) • Notes + CYU begins (~12 min) **Materials:** One triangle manipulative set per group (print on card stock — see template in downloads) • Boards and markers • Protractors and rulers for Task 2 **Nav instrument rows active:** Rows 1–2 spiral • Row 3 (Triangle Congruence Shortcuts) activates today **Manipulative prep:** One triangle printed and traced on card stock per group — five sets suggested (see note below). Cut out side-length strips and angle cutouts. Label each angle piece clearly.

OBJECTIVES

By the end of this lesson students can:

- Define triangle congruence in terms of all six corresponding parts.
- Identify which shortcuts (SSS, SAS, ASA, AAS, HL) guarantee congruence and explain why each works.
- Explain why SSA is not valid and demonstrate this with a physical counterexample.
- Explain why AAA is not valid using the parallel line construction.
- Distinguish between AAS (valid) and SSA (not valid).

Narrative Arc — Open class — before Task 1

"Last class, we proved that an equilateral triangle can exist in our system. We proved it by constructing one. Today we ask a different question: when are two triangles exactly the same? And — how much information do you actually need to check?"

MANIPULATIVE NOTE

You do not need five different triangles

One triangle printed and traced onto card stock works perfectly well. The variety across five sets was intended to show students that the shortcuts are not specific to any particular triangle — but this universality can be stated rather than demonstrated physically. Students accept it readily. One set per group is sufficient.

Five suggested sets if using variety: (1) scalene 5-6-7, angles ~44°, 62°, 73° (2) isosceles 7-7-5.1, angles 50°-80°-50° (3) right 3-4-5, angles 90°-37°-53° (4) scalene 6.6-5.8-6, angles 60°-70°-50° (5) obtuse 8-6-11.3, angles 120°-25°-35°

TASKS

Task 1 — What Does It Mean for Two Triangles to Be the Same? (≈10 min)

Before distributing any materials, ask groups to write and discuss:

"What would it mean for two triangles to be exactly the same? What would have to be true?"

What to listen for

Groups will land on: same side lengths, same angles, same shape. Push toward precision: which parts? How many? Guide them to: two triangles are congruent when all six corresponding parts match — three pairs of sides and three pairs of angles. Emphasize the word corresponding: it is not just that the same measurements appear somewhere, but that they appear in matching positions.

Once the class agrees: "So congruence requires all six parts. But do we always need to check all six? That is what you are going to find out."

Task 2 — The Minimum Information Problem (≈35 min)

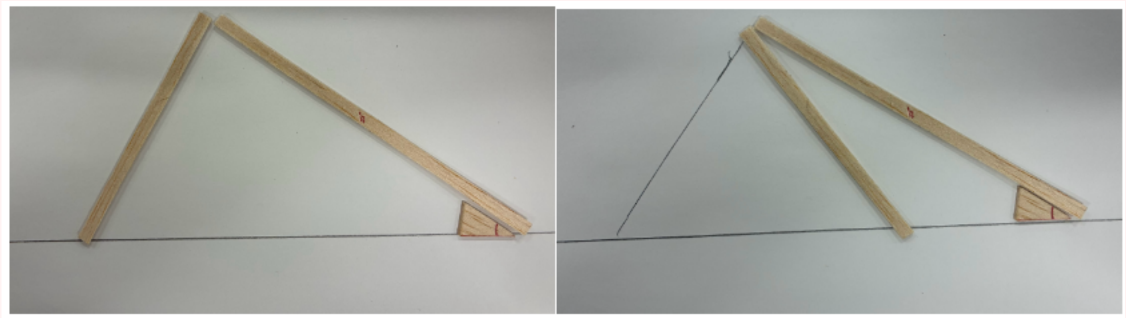
Distribute one set of manipulatives per group. Say:

"Your set contains the side lengths and angles of one specific triangle. Find the minimum number of pieces — and which combinations — that are enough to guarantee exactly one triangle. For each combination: form a conjecture first, then try to find a counterexample. If you can find a different triangle using the same pieces, the combination fails. If you cannot find one after serious effort, that is evidence it might work — but failing to find a counterexample is not a proof. We will need to accept these carefully."

⚠️ SSA — The Critical Demonstration (Do Not Skip)

Most students will try SSA and only build one triangle, concluding it works. The failure is genuinely hard to see without a physical reference.

The reference line technique: draw or tape a straight baseline on paper. Place the angle piece at one endpoint. The second side-length strip pivots freely from the angle vertex. Students can see that the strip can reach the baseline in two different places — two non-congruent triangles, same SSA information.



Run this demonstration for the whole class before they test SSA. Without it, groups that happen to build it one way will conclude SSA works.

AAA — Surface the geometric argument

Students will likely feel AAA fails intuitively (‘same shape, different size’). Push for the geometric argument: draw triangle ABC. Now draw DE parallel to BC, with D on AB and E on AC. By corresponding angles (Unit 2), ∠ADE = ∠ABC and ∠AED = ∠ACB. Triangle ADE has all three angles equal to triangle ABC but is clearly smaller. One counterexample disproves the general claim — a direct callback to Unit 2.

AAS vs. SSA — Address directly

Both involve two angles (or two sides) plus one additional piece. Address the distinction after students have tested both:

***AAS works:** knowing two angles forces the third (angle sum = 180°), reducing AAS to ASA. The angles do the constraining; the side just locates the triangle.*

***SSA fails:** the third angle is not forced. The second side can swing to two positions. Nothing prevents ambiguity.*

CONSOLIDATION

Run a teacher-curated gallery walk first. Identify: a board correctly reasoning about a valid shortcut, a board wrestling with SSA, a board with the AAA intuition but not yet a geometric argument. Direct student attention deliberately — this is not a free browse.

Bring out in order:

1. Name and define all valid shortcuts together

For each shortcut, ask a group to state why it works, not just that it does. Goal: students articulate the constraint that pins the triangle uniquely.

2. Address SSA as a class

Run the reference line demonstration for the whole class if needed. Ask: 'If SSA doesn't work, what is missing? What would need to be different?' Guide toward SAS as the repaired version.

3. Build the AAA argument together

Walk through the parallel line construction. Ask students to write it as a conditional statement. Confirm: one valid counterexample is sufficient to disprove a general claim. Connect explicitly to Unit 2.

4. Name AAS vs. SSA side by side on the board

Write both. Ask: 'One works, one doesn't. What is different?' Let students articulate it. Then confirm: in AAS the angles constrain; in SSA they do not.

Epistemological honesty note — say this to students

SSS, SAS, ASA, AAS, and HL can be proven. In Euclid's Elements, SAS is Proposition 4, SSS is Proposition 8, and ASA/AAS are Proposition 26. These are earned results, not starting assumptions. Euclid's proofs of these involve superposition — placing one triangle on another — which requires assumptions Euclid did not explicitly list. Hilbert later identified this gap. By accepting these as postulates, we sidestep a thorny issue and gain tools to practice proof. We are not ignoring the gap — we are making a deliberate choice, and we name it honestly.

DEFINITIONS AND KEY POINTS REFERENCE

Triangle Congruence — Definition

Two triangles are congruent if and only if all six corresponding parts are congruent: three pairs of sides and three pairs of angles.

Written as a biconditional: Two triangles are congruent ⇔ all corresponding sides are congruent AND all corresponding angles are congruent.

Correspondence order matters. In $\triangle ABC \cong \triangle DEF$: $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$. Writing $\triangle ABC \cong \triangle EDF$ makes a different claim. Emphasize this — it will cause errors in proof writing if not addressed now.

Shortcut	What it requires	Why it works / why it fails	Valid?
SSS	All three pairs of corresponding sides congruent	Three fixed side lengths determine a unique triangle — the shape cannot change without changing a side. No ambiguity.	✓
SAS	Two pairs of sides and the included angle congruent (angle is between the two sides)	The included angle pins the two sides together. The third side and remaining angles are forced.	✓

ASA	Two pairs of angles and the included side congruent (side is between the two angles)	Two angles fix the shape; the side between them fixes the scale. Everything else is determined.	✓
AAS	Two pairs of angles and a non-included side congruent	Two angles force the third (angle sum = 180°), reducing AAS to ASA effectively.	✓
HL	Right triangles only: hypotenuse and one leg congruent	Right angle gives one angle pair; HL gives two side pairs. Follows from AAS — the right angle provides the missing angle.	✓ (right △ only)
SSA	Two pairs of sides and a non-included angle	The second side can swing to two positions. Two non-congruent triangles can share the same SSA.	✗
AAA	All three pairs of angles congruent	Angles fix shape but not size. Draw DE ∥ BC inside △ABC — △ADE has the same angles but is smaller.	✗

CPCTC — Introduction and Course Policy

CPCTC: Corresponding Parts of Congruent Triangles are Congruent.

Written as a biconditional: if two triangles are congruent, then all six corresponding parts are congruent. AND if all six corresponding parts of two triangles are congruent, then the triangles are congruent.

Course policy: Before writing CPCTC as a justification in any proof, board work, quiz, or assessment, you must first write its full meaning: "Corresponding Parts of Congruent Triangles are Congruent: if two triangles are congruent, then all six corresponding parts (three sides and three angles) are congruent." The abbreviation is only permitted after the definition has been written.

CPCTC is not a new idea — it is the definition of congruence students established in Task 1 of this lesson. Name this explicitly. Students were using it as a label without making the connection.

STUDENTS MAKE NOTES ON

✎ **Students make notes on:**

- Triangle congruence definition as a biconditional
- Shortcut table: SSS, SAS, ASA, AAS, HL, SSA, AAA — what each requires, why it works or fails, valid or not
- SSA failure: diagram showing two non-congruent triangles from the same SSA information, written argument
- AAA failure: parallel line construction argument written as a conditional statement
- CPCTC: full definition, biconditional form, and course policy

Connection Points	• Draws from Unit 2: one counterexample disproves a general claim — used for both SSA and AAA

	• Draws from Unit 2: the parallel line construction uses alternate interior angles from the transversal lesson • Feeds into Lesson 3.3: CPCTC policy is in force from here onward; correspondence order matters in every diagram • Feeds into Lesson 3.5: shortcuts are the tools students draw from in every proof they write