

## Visual Reasoning

Unit 0: Problem Solving and Mathematical Habits • Day 4 of 4

### At a Glance

**Pacing:** ~45 min tasks (two tasks, back to back) • ~45 min consolidation + group assessment

**Materials:** Boards and markers • Dot paper or grid paper helpful for visual proofs

**Mathematical habits revisited today:** All eight steps from Lesson 0.3 — visual reasoning added as an additional tool

**Group assessment:** Randomly select 3 groups to assess on the group work rubric. All groups self-assess. Compare results.

### OBJECTIVES

By the end of this lesson students:

- Have used visual models to discover and justify mathematical conjectures.
- Understand that a visual argument can be a proof — not just an illustration.
- Can apply all eight mathematical thinking habits to a new problem.

### TASKS

#### Task 1 — The Mountain Sum (≈25 min)

##### Prompt

What happens when you add all consecutive numbers up to a point and then back down? For example:  $1+2+3+4+5+4+3+2+1$ . Try a few. Make a conjecture as soon as you notice anything.

Teacher questions:

- What do you notice about the results?
- What does it mean for a number to be a perfect square?
- Can you draw a picture of the sequence? Can you see the square in the picture?
- Does your conjecture work for  $n = 12$ ?
- Are there any counterexamples?

##### Key conjecture and visual

The sum  $1+2+\dots+n+\dots+2+1 = n^2$ . Show a square of  $n^2$  dots arranged in an  $n \times n$  grid. Ask students to identify the mountain sum in the diagonals of the square.

**Extension:** What is the sum of just the numbers up to  $n$  (not back down)? How is it related to the mountain sum? Guide toward:  $\text{sum to } n = (n^2 + n) / 2$ .

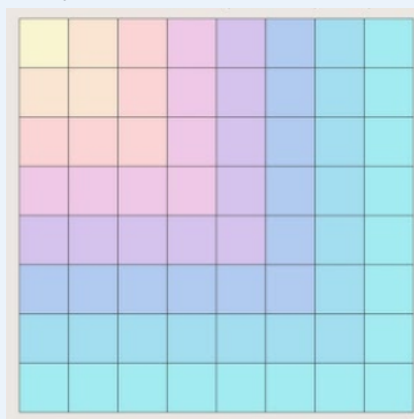
## Task 2 — Sum of Consecutive Odd Numbers ( $\approx 20$ min)

### Prompt

**Add consecutive odd numbers starting from 1. Example:  $1+3+5$ . Do this for several sums. Form a conjecture as soon as you notice anything. Then try to create a visual argument — not just an algebraic one.**

### Key conjecture and visual

$1+3+5+\dots+(2n-1) = n^2$ . *The visual: each new odd number can be drawn as an L-shaped border (gnomon) around the previous square. Adding the next odd number adds one more L-shape, expanding the square by one row and column. The  $n$ th L-shape represents the  $n$ th odd integer. The result is always a perfect square.*



Extension:

- Find the sum of the first  $n$  even numbers. (Answer:  $n(n+1)$ . Visual: add a column to the odd-number square to create an  $n \times (n+1)$  rectangle.)
- Use the even number result to verify the sum-to- $n$  formula from Task 1 extension.

## CONSOLIDATION

Today's reflection should emphasize the power of visualization as a tool to: organize thinking, discover patterns, and build convincing arguments. The point of both tasks was not to memorize formulas but to demonstrate that geometry and visual reasoning can prove algebraic results.

### The key message to state explicitly

*A visual argument is a proof. Not every proof needs symbols and algebra. The L-shaped gnomon argument for the sum of odd numbers is a complete, rigorous proof. The diagram is the proof.*

*This is the last lesson of Unit 0. Over four days students have built the habits of mind that the entire course depends on. Name what they have: specializing, organizing, generalizing, testing, conjecturing, justifying, convincing, and now visual reasoning. These are not geometry skills — they are mathematical thinking skills that transfer to any domain.*

Have students share discoveries or breakthroughs. Ask: where did you get stuck, and how did you get unstuck? What technique did you try that did not work — and why was trying it still worthwhile?

▶ **Narrative Arc** — End of Unit 0 — transition to Unit 1

*“Over these four days you have done mathematics. Not followed procedures — done mathematics. You have tried things, organized results, looked for patterns, made conjectures, and started building arguments for why things must be true.”*

*“Unit 1 starts next class. The content becomes geometric — definitions, points, lines, distances, angles. But the way you work doesn’t change. You will still start by trying something. You will still ask whether it always works. You will still look for the argument that makes it undeniable.”*

**Feeds into**

- Unit 3: visual reasoning returns as a proof technique — diagrams are not just illustrations
- Unit 6: visual reasoning appears in the scaling law proofs (area as a square of the scale factor)
- Throughout: the habit of asking ‘can I see why this is true?’ is one of the most valuable mathematical instincts