Lesson 8.5 — Teacher Reference Notes

Tangent Lines and the Chord-Bisector Theorem

Unit 8: Circles • Day 5 of 7

At a Glance	
	Pacing: Launch — small shard challenge (~8–10 min) • Thinking task — finding the center (~25–30 min) • Gallery walk (~5 min) • Consolidation — chord-bisector + radius-tangent (~20–25 min) • Notes (~10 min) • CYU (~10–12 min) **Materials:** Compasses, rulers, • The small shard for today's **Nav instrument row active:** Row 5 (Tangent lines and chord theorems → Consequences of the definition) activates today. Homework spirals across Rows 1–5. **Key setup note:** Prepare a small shard per group — fragment covering less than a semicircle (20°–130°). Cut so there is no way to fit a right-angle corner across the arc; the Thales trick must fail. If you use the same radius as the Lesson 8.4 shard, groups can check today's circumference against yesterday's — a powerful verification.

OBJECTIVES

By the end of this lesson students can:

- State the chord-bisector theorem and prove it using the perpendicular bisector theorem from Unit 4.
- Use the chord-bisector theorem to find the center of a circle from any fragment by constructing the perpendicular bisectors of two chords.
- State the radius-tangent theorem and prove it using the shortest-distance argument.
- Apply both theorems to standard problems involving chords, tangents, and external points.
- Identify the single underlying fact — the center is equidistant from every point on the circle — that connects both theorems and several others in the unit.

▶ Narrative Arc — Open class with this

“Yesterday you solved the shard problem for a large fragment using Thales' theorem. We promised we would finish the harder case (any fragment, no matter how small) once we had one more tool. Today is that lesson. And it turns out the same tool gives us tangent lines too. Two big theorems from one underlying idea we learned in lesson 8.1.”

LAUNCH — THE SMALL SHARD

Launch (≈8–10 min)

Give each group the small shard. Let them try the paper-corner trick from Lesson 8.4 first. They will quickly find it does not work — the fragment is too small to span a right angle. Then say:

“In Lesson 8.4 we solved the shard problem for a large fragment using Thales' theorem. That trick fails here. This is the version of the problem we said we couldn't do yet. Now we have more tools. Can you find the center of the original circle using only this fragment?”

Send groups to the boards. Do not hint further.

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TASK — FINDING THE CENTER

What you will see (in order of sophistication):

Stage	What groups do	How to push
1. Drawing chords	Mark two or three points on the arc, connect them with chords	*"Good. Now what do you know about the center and those chord endpoints?"*
2. Trying perpendiculars	Draw perpendiculars to the chords, possibly not at the midpoints yet	*"Does the perpendicular have to go through the midpoint? Why? What theorem from Unit 4 is relevant?"*
3. Getting the bisectors	Construct the perpendicular bisectors of two chords, find their intersection	*"Why must that intersection point be the center? Write a justification."*
4. Finishing the problem	Measure a radius from the found center to any point on the arc, calculate circumference	*"How confident are you that you have the true center, not just an estimate? What makes you certain?"*

The key nudge — perpendicular bisector theorem from Unit 4

The key nudge at stage 2 is reminding students of the perpendicular bisector theorem from Unit 4: the perpendicular bisector of a segment is the locus of all points equidistant from the two endpoints. The center is equidistant from any two points on the circle (all radii are equal), so it must lie on the perpendicular bisector of any chord. Two chords give two perpendicular bisectors; their intersection is the unique point on both — the center.

Extension Ladder

Level 1: You used two chords to find the center. Would one chord be enough? What is the minimum number of chords you need, and why?

Level 2: A tangent line touches a circle at a single point. Draw a line that appears to be tangent to your circle at a point on the arc. What do you think is the relationship between the radius to that point and the tangent line? Try to justify it.

Level 3: From a point P outside your circle, draw two lines that each touch the circle at exactly one point. Measure the two tangent segments from P. What do you notice? Can you prove it?

Level 4 — Open: The chord-bisector theorem and the radius-tangent theorem both follow from the same underlying fact about circles. Can you state what that fact is? Can you find any other theorems in this unit that also follow from it?

CONSOLIDATION — TWO THEOREMS FROM ONE IDEA

Open by asking a group that succeeded with the shard to explain their method. Let the class verify that it works. Then ask: "Why does it work? What theorem are you using?" Surface the chord-bisector theorem from student language before writing the formal version.

✎ **Students make notes on:**

- Chord-bisector theorem: statement, diagram, and proof in their own words — explicitly naming the Unit 4 theorem used.
- Radius-tangent theorem: statement, diagram, and proof in their own words — emphasizing that the proof uses only the definition of tangency, not angles or arcs.

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- The construction: how to find the center of a circle from a fragment, with the reason it needs two chords (one chord gives only a line).
- The single underlying fact — the center is equidistant from every point on the circle — stated as a 'big idea' in their own words, with at least two theorems flowing from it.
- How today connects back to Unit 4 (perpendicular bisectors, triangle congruence) and to Lesson 8.1 (what a circle is).

Chord-Bisector Theorem

The perpendicular bisector of any chord of a circle passes through the center.

Proof: Let AB be a chord of a circle with center O.

OA = OB (both radii).

By the perpendicular bisector theorem (Unit 4), any point equidistant from A and B lies on the perpendicular bisector of AB.

Therefore O lies on the perpendicular bisector of AB. ∎

Ask: “Which theorem from Unit 4 made this proof possible?” Make sure students name it explicitly. Then ask: “What made OA = OB?” The definition of a circle — all radii are equal. The definition is doing the work again.

Radius-Tangent Theorem

The radius to a point of tangency is perpendicular to the tangent line.

Proof: Let T be the point of tangency and O the center.

Let X be any other point on the tangent line.

Since the tangent meets the circle at only T, the point X is outside the circle.

Therefore $OX > r = OT$.

So OT is the shortest distance from O to the tangent line.

The shortest distance from a point to a line is always perpendicular.

Therefore $OT \perp$ tangent line. ∎

Ask: “What does this proof use about the tangent line?” It uses only the definition of tangency — that the tangent touches at exactly one point, forcing every other point on the line to be outside the circle. The proof uses no angle relationships, no arc measures, nothing from the inscribed angle theorem. It is a purely definitional argument.

Bonus — Equal tangent segments from an external point

If a group found this in the extension (Level 3): from an external point P, draw tangents to the circle touching at T_1 and T_2. Then $PT_1 = PT_2$. Proof: $OT_1 = OT_2$ (radii), OP = OP (shared), and $\angle OT_1P = \angle OT_2P = 90°$ (radius-tangent). So, triangles OT_1P and OT_2P are congruent by hypotenuse-leg, giving $PT_1 = PT_2$. Worth five minutes if a group found it — a satisfying application of Unit 3 triangle congruence inside circle geometry.

The unifying idea — do not skip this

Close consolidation by asking Extension Level 4 directly to the whole class: “Both theorems today came from the same fact about circles. What is it?”

The center of a circle is equidistant from every point on the circle.

This single fact — which is just the definition of a circle — implies:

- The perpendicular bisector theorem (Unit 4) → chord-bisector theorem
- The shortest-distance argument → radius-tangent theorem
- All radii equal → isosceles triangles in proofs throughout the unit

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The definition of a circle is not just a starting point. It is the engine that drives the entire unit.

⚠️ **Why this consolidation moment matters**

This is one of the most important moments in the unit to be explicit about. Students often experience geometry as a collection of disconnected theorems. The point of this course is to see theorems as a connected web of consequences. The definition of a circle is the root of that web. Every major result in Unit 8 traces back to it. Naming that explicitly, and asking students to find the chain of reasoning, is the highest-level thinking the unit asks for.

▶️ **Narrative Arc — End of class — transition to Lesson 8.6**

"Four lessons in. You have the equation of a circle. You have similarity. You have inscribed angles. You have tangents and chords. Tomorrow we close a loop you didn't know was open. In Unit 7 you learned the Law of Sines and used it to solve triangles. But there was a question we never answered: what is that common ratio a/sin A? It has a value. Tomorrow we find out and the answer is in a circle."

Connection Points	**Draws from Unit 4:** The perpendicular bisector theorem (Unit 4 was about triangle theorems and perpendicular bisectors) is the central engine of the chord-bisector proof. Students who remember Unit 4 should find this proof almost immediate. The satisfaction is in recognizing an old tool doing new work. **Draws from Unit 3:** The equal tangent segments result (if reached) uses hypotenuse-leg congruence from Unit 3. **Draws from Lesson 8.1:** The definition of a circle (all radii equal) is the single fact that makes OA = OB in the chord-bisector proof and that makes 'outside the circle' mean 'distance > r' in the radius-tangent proof. **Sets up Lesson 8.6:** The circumscribed circle of a triangle — the unique circle through all three vertices — is found using exactly the chord-bisector construction students learned today. Without today's construction, tomorrow's setup would be opaque.

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