Lesson 1.2 — Teacher Reference Notes

Distance and Midpoint in the Plane

Unit 1: Foundations of Geometry • Day 3 of unit

At a Glance	**Pacing:** Google Maps activator (5 min, resolution saved for end) • ~45 min task • ~20 min notes (three formulas) • ~20 min activator resolution + proportional reasoning **Materials:** Boards and markers • Google Maps screenshots projected (start and end coordinates provided below) **Nav instrument rows active:** Rows 1–2 spiral • Row 3 (Distance and Midpoint) activates today **Save the activator resolution:** Students who have something to check their answer against are more motivated to do the proportional reasoning calculation. Hold it until the end.

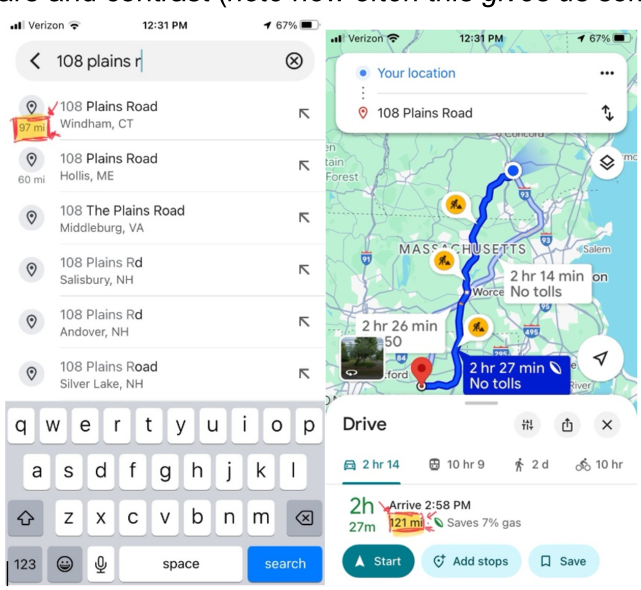
OBJECTIVES

By the end of this lesson students:

- Can calculate the Euclidean distance between two points using the distance formula.
- Can find the midpoint between two points including in different quadrants.
- Can explain how the distance formula is the Pythagorean theorem in coordinate form.
- Can use proportional reasoning to convert between coordinate degrees and real-world distance.

ACTIVATOR

Project Google Maps showing two locations with the as-the-crow-flies straight-line distance visible. Say "I noticed something strange seems to happen when I ask google maps for the directions to a place. When I type in the address, I get a screen like the left, but then when I click where I want to go, I get a screen like the right. Compare and contrast (*note how often this gives us something to explore).*



Coordinates for the activator calculation (use at end of class)

Start: 42.99027° N, 71.4243° W — End: 41.69886° N, 72.16748° W

One degree of latitude ≈ 69 miles. One degree of longitude at ~42° N ≈ 53 miles. Subtracting coordinates: ~89 miles south, ~39 miles west. Distance formula gives $\sqrt{(89^2+39^2)} \approx 97$ *miles — consistent with what the map shows.*

Name the proportional reasoning explicitly: students will use it constantly in the similarity unit. This is their first encounter with it in a context that gives it genuine motivation.

TASK

The Pregnant Passenger (≈40 min)

Prompt

You are a taxi driver in a city on a perfect grid. Your passenger is about to have a baby and needs to reach the hospital at H(8, 6) from your current location at S(2, 4) as fast as possible. You can only travel along grid lines — no diagonals. What is the quickest path?

Keeping-thinking questions:

- How do you know that is the quickest path?
- Are there other paths that are equally fast?
- Can you apply your rule to two other locations?
- Can you generalize: what is the taxicab distance between any (x_1, y_1) and (x_2, y_2)?

Essential extension — from roads to birds (give to all groups once they have the taxicab formula)

- Draw the shortest straight-line path between S and H. Calculate its length. What geometric shape is hiding in the formula?
- Write a general rule for the straight-line distance between any two points.
- Find the midpoint between S and H. How do you know it's the midpoint? Use the distance formula twice.

Teacher note: start with a 3-4-5 triangle (e.g., (4, −2) and (7, 4)) before generalizing. The hidden right triangle — horizontal leg, vertical leg, hypotenuse — is the Pythagorean theorem in coordinate form. Make this connection explicit in consolidation and name it as a pattern that will recur throughout the course.

⚠️ **Watch for: taxicab formula in Euclidean problems**

Students who find the taxicab formula satisfying will sometimes apply it later where the Euclidean formula is needed. Name this directly in consolidation: taxicab geometry was our entry point for thinking about distance, but the formula we use going forward is the Pythagorean one. The hidden right triangle is the key.

NOTES (≈20 MIN)

After the board work, students write their own notes on the three formulas before checking them against the class record. This begins the habit of making meaningful notes from their own work rather than copying from the board.

The hidden triangle callout — say this explicitly

The hidden right triangle in the distance formula is the first appearance of a pattern that will run through the entire course. Triangles are everywhere in geometry — hiding inside quadrilaterals, circles, similarity arguments, and the distance formula itself. When students learn to look for the hidden triangle, they have one of the most powerful problem-solving tools in the course. Name it here. We will name it again.

✎ **Students make notes on:**

- Distance formula with diagram showing the hidden right triangle and its connection to the Pythagorean theorem
- Midpoint formula with worked example
- Proportional reasoning example from the activator resolution

FORMULAS REFERENCE — TEACHER COPY

Taxicab Distance Formula

$$d_t = |x_2 - x_1| + |y_2 - y_1|$$

Sum of the absolute horizontal and vertical changes. Works only along grid lines — no diagonals. Students find this first in the Pregnant Passenger task. Name it explicitly and distinguish it from the Euclidean formula before closing consolidation.

Euclidean Distance Formula — show this as the Pythagorean theorem

$$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

When going over this with students, draw it as a right triangle first: the horizontal leg is ($x_2 - x_1$), the vertical leg is ($y_2 - y_1$), and the hypotenuse is d. Then write the Pythagorean theorem: $a^2 + b^2 = c^2$. Substitute the legs and solve for c. The distance formula falls out directly. Students who see this derivation understand why the formula works and will not need to re-memorize it.

Worked example: distance from (2, 4) to (8, 6). Horizontal leg = 8 − 2 = 6. Vertical leg = 6 − 4 = 2. d = $\sqrt{(6^2 + 2^2)} = \sqrt{(36 + 4)} = \sqrt{40} = 2\sqrt{10} \approx 6.32$

Midpoint Formula

$M = ((x_1 + x_2)/2,\ (y_1 + y_2)/2)$

Average of the x-coordinates, average of the y-coordinates. Ask students why averaging works: the midpoint is equally far from each endpoint, so it sits at the mean of each coordinate.

Worked example: midpoint of (2, 4) and (8, 6). M = ((2+8)/2, (4+6)/2) = (5, 5). Verify by checking distances: distance from (2,4) to (5,5) = $\sqrt{(9+1)} = \sqrt{10}$. Distance from (5,5) to (8,6) = $\sqrt{(9+1)} = \sqrt{10}$. Equal — confirmed midpoint.

Finding a missing endpoint: if M is the midpoint of A and B, and M and A are known, solve the midpoint formula for B. Example: M = (5, 5), A = (2, 4). Then $5 = (2 + x_2)/2 \rightarrow x_2 = 8$. And $5 = (4 + y_2)/2 \rightarrow y_2 = 6$. So B = (8, 6).

Proportional Reasoning — activator resolution

Start: 42.99027° N, 71.4243° W — End: 41.69886° N, 72.16748° W

Difference in latitude: 42.99027 − 41.69886 = 1.29141° × 69 mi/° ≈ 89.1 mi south.
Difference in longitude: 72.16748 − 71.4243 = 0.74318° × 53 mi/° ≈ 39.4 mi west.
Straight-line distance: $\sqrt{(89.1^2 + 39.4^2)} \approx \sqrt{(7938.8 + 1552.4)} \approx \sqrt{9491.2} \approx 97.4$ miles.

Name the proportional reasoning move explicitly: 1 degree of latitude = 69 miles is a unit conversion ratio. Multiplying a degree value by the ratio gives miles. This is proportional reasoning — the same structure students will use constantly in the similarity unit.

Connection Points	• Draws from Unit 0: proportional reasoning appears in the closing activator • Feeds into Lesson 1.3: distance formula is used for slanted-side perimeter • Feeds into Unit 6: proportional reasoning returns as the engine of similarity; the hidden triangle pattern is named again in Unit 5–7