Unit 7

Trigonometry

Invariant Ratios, Geometry's Reach, and the Shape of Sine

13 days including assessment • CYU replaces homework throughout

Unit Navigation Instrument *Big Idea: When one acute angle in a right triangle is fixed, the ratios of its sides become invariant. Those invariant ratios are functions of the angle — and by creating right triangles inside larger triangles, we can extend that reasoning to any triangle.*

Subtopic	Basic	Intermediate	Advanced
Trig Ratios *→ Discovering the invariants*	Define sine, cosine, and tangent. Write the three ratios correctly for a given triangle.	Explain why trig ratios are invariant using similarity. Identify which ratio to use in a given situation.	Explain why fixing one acute angle fixes all three ratios. Connect the invariance of trig ratios to the AA similarity argument from Unit 6.
Solving for Missing Sides *→ Using invariants*	Use sine, cosine, or tangent to find one missing side.	Choose the correct ratio and solve multi-step problems including angle of elevation and depression.	Model and solve real-world problems using trig. Justify each step, naming the ratio used and why it applies.
Solving for Missing Angles *→ Inverting the function*	Use inverse trig to find a missing angle given two sides.	Use inverse trig in multi-step problems. Distinguish when to use trig vs. inverse trig from problem structure.	Explain why inverse trig works as a function. Connect to trig as a function and its domain restriction for acute angles.
Special Right Triangles *→ Exact values worth deriving*	Fill in all side lengths for 45-45-90 and 30-60-90 triangles given the angles.	Use special right triangle ratios to find exact side lengths. Rationalize denominators.	Reconstruct the derivation of both special right triangle types from first principles. Verify the Pythagorean Identity using exact values.
Estimation *→ Trig as a function*	Use special right triangle anchors to estimate trig values. State whether sine and cosine increase or decrease over 0° to 90°.	Describe how sine and cosine change as angles increase. Explain geometrically why they move in opposite directions.	Reason about whether a calculated answer is plausible. Sketch $y = \sin\theta$ and $y = \cos\theta$ on the same axes and explain the crossing point.

Trig Beyond Right Triangles *→ Extending invariants*	Apply the Law of Sines and Law of Cosines correctly.	Determine which law applies in a given situation and explain the reasoning.	Derive the Law of Sines from area formulas. Explain why the Law of Cosines reduces to the Pythagorean theorem when C = 90°.

What This Unit Is About

Trigonometry is not a detour from geometry. It is one of geometry's most important consequences. Students entering this unit have already established similarity, proportional reasoning, and the idea that fixing one angle in a right triangle determines a unique shape. Trigonometry is what happens when you name the ratios that similarity creates and treat them as functions of the angle.

The unit is built in three phases. The first phase — Lessons 7.1 through 7.3 — discovers, formalizes, and inverts the three basic trig ratios. The second phase — Lessons 7.4 and 7.5 — establishes exact values through geometric proof and briefly connects to the graphical representation of sine and cosine. The third phase — Lessons 7.6 and 7.7 — extends trig beyond right triangles through the Law of Sines and Law of Cosines, culminating in the Everest problem and a problem requiring tools from every unit of the course.

This unit was taught and reflects those experiences honestly. Several classroom stories are preserved here — including moments where lessons landed in one class and failed in another. That is normal. The same lesson, the same materials, and the same teacher can produce dramatically different outcomes depending on where students are, what energy they bring, and whether they feel ownership of the work. Monitoring, reading formative feedback, and adjusting are not emergency responses. They are the constant practice of teaching.

A recurring flag: algebra in CYU problems

Across this unit — and the derivation of the Law of Cosines in particular — algebraic errors consistently disrupted otherwise correct mathematical thinking. Students expanded $(b-x)^2$ *as* b^2-x^2*. They applied order of operations incorrectly to expressions like* $240-236\cdot\cos(15°)$*.*

These are not trigonometry failures. They are algebra gaps that trig exposed. The lesson-level rewrites now address specific errors at the point where they surface, but the *structural fix is to include at least one algebra problem in every CYU set from Unit 1 onward — spiral practice that keeps algebraic fluency active throughout the year.*

A Note on Lesson 7.5

Lesson 7.5 — The Shape of Sine — completes the functional picture of right triangle trigonometry by connecting the table, formula, and verbal definition students have built to the graphical representation. It is a worthwhile lesson when time allows. However, it sits at the boundary between geometry and precalculus. The full story of sine and cosine as continuous periodic functions belongs in precalculus, where the unit circle extends their domain beyond 90°. Lesson 7.5 offers a preview of that story — the shape of the curve from 0° to 90°, the Desmos moment that plants a seed — without completing it.

Teachers who are behind schedule should not feel compelled to force this lesson. The unit is coherent without it. Teachers who have time and whose classes are ready will find it a satisfying moment of synthesis. The lesson is included in full in the lesson materials; the decision of whether to teach it belongs to the teacher reading this.

Lesson Sequence Overview

Lesson 7.1: Ratios That Refuse to Change — families of similar right triangles, invariant ratios discovered, three ratios named, class trig table built (1 day)

Lesson 7.2: If We Can Measure the Angle, We Can Measure Anything — solving right triangles, clinometer design and build, field measurement (2 days)

Lesson 7.3: Choosing the Steepest Safe Path — avalanche decision task, inverse trig introduced, ADA ramp compliance, mixed practice (2 days)

Lesson 7.4: Triangles Worth Remembering — 45-45-90 and 30-60-90 derived from first principles, exact values, Pythagorean Identity (2 days)

Lesson 7.5: The Shape of Sine — graphical representation, four function representations connected, Desmos moment (1 day, optional)

Lesson 7.6: The Law of Sines — area from two sides and included angle, Law of Sines derived, Mount Everest problem (2 days)

Lesson 7.7: The Law of Cosines — derived from the Pythagorean theorem, special cases, ambiguous case, the course converges (2 days)

Lesson 7.8: Assessment