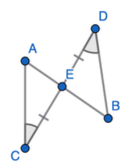
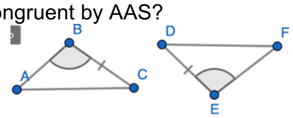
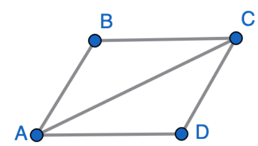
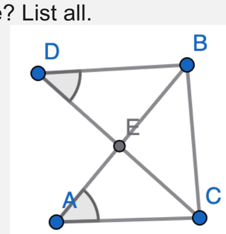
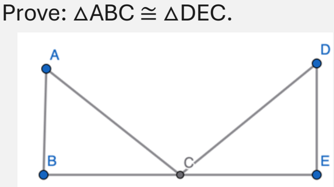
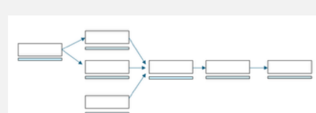
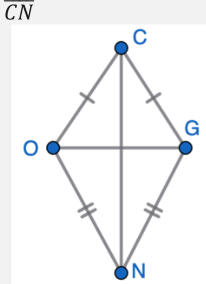
Subtopic	Basic	Intermediate	Advanced
Row 1 Constructions *→ Proof begins with what we can construct*	1. Construct (no measurement) a midpoint of the line segment.	2. Justify each step you made in your construction for problem 1.	3. Prove that the point your constructed is actually a midpoint.
Row 2 Postulates vs. Propositions *→ What we accept vs. what we must earn*	4. Compare and contrast postulate and proposition.	5. Look at the proof of proposition 2 below. Why could this have not been the first proposition Euclid proved in Elements?	6. Assuming all laws of logic are followed correctly, how could deductive reasoning produce a false result?
Row 3 Congruence Shortcuts *→ The geometric facts we draw from*	7. Which shortcut proves these congruent.  8. What would be needed to be given to prove the triangles congruent by AAS? 	9. Construct an image and use it to argue that AAA does not prove two triangles are congruent. You may not use any measurements.	10. Given: $\overline{BC} \| \overline{AD}$ and $\overline{AB} \| \overline{CD}$ prove: $\triangle ABC \cong \triangle CDA$ 
Row 4 Reading a Diagram *→ Given vs. earned from the diagram*	11. What things can you tell me must be true, from the diagram alone? List all. 	12. Name one bit of additional information you would need to prove: $\triangle DBC \cong \triangle ACB$ in number 11.	This will be assessed in other proofs.
Row 5 Proof Structure *→ How each step earns the next*	This will be assessed in other proof problems.		
Row 6 Writing Complete Proofs *→ A complete, unjustifiable argument*	13. Fill in the blanks on the following proof: Given: BA ⊥ BE, DE ⊥ BE, C is the midpoint of BE, and ∠DCB ≅ ∠ECA. Prove: △ABC ≅ △DEC. 	14. Fill in the flow chart to prove: Given: trapezoid ABCD with $\overline{AD} = \overline{BC}$. Prove $\overline{AE} = \overline{BE}$ 	15. Given the diagram, prove $\overline{OG} \perp \overline{CN}$ 

<table>
<tr><th>Subtopic</th><th>Basic</th><th>Intermediate</th><th>Advanced</th></tr>
<tr><td></td><td>
<table>
<tr><td>Statement</td><td>Justification</td></tr>
<tr><td>BA ⊥ BE
DE ⊥ BE</td><td></td></tr>
<tr><td></td><td>Definition of ⊥</td></tr>
<tr><td>∠B ≅ ∠E</td><td></td></tr>
<tr><td></td><td>given</td></tr>
<tr><td>$\overline{BE} \cong \overline{EC}$</td><td></td></tr>
<tr><td>∠DCB ≅ ∠ECA</td><td></td></tr>
<tr><td>(2 statements)</td><td>Angle addition postulate (CN5)</td></tr>
<tr><td>∠DCA + ∠ACB ≅ ∠DCA + ∠DCE</td><td></td></tr>
<tr><td>∠ACB ≅ ∠DCE</td><td></td></tr>
<tr><td>△ABC ≅ △DEC.</td><td></td></tr>
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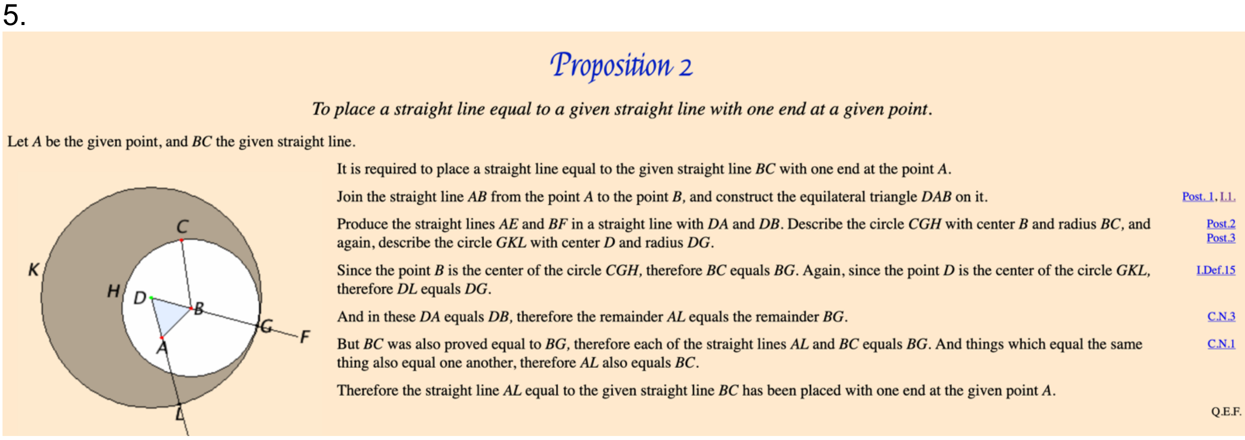
5.



Proposition 2

To place a straight line equal to a given straight line with one end at a given point.

Let *A* be the given point, and *BC* the given straight line.

It is required to place a straight line equal to the given straight line *BC* with one end at the point *A*.

Join the straight line *AB* from the point *A* to the point *B*, and construct the equilateral triangle *DAB* on it. Post. 1, I.1.

Produce the straight lines *AE* and *BF* in a straight line with *DA* and *DB*. Describe the circle *CGH* with center *B* and radius *BC*, and again, describe the circle *GKL* with center *D* and radius *DG*. Post.2 Post.3

Since the point *B* is the center of the circle *CGH*, therefore *BC* equals *BG*. Again, since the point *D* is the center of the circle *GKL*, therefore *DL* equals *DG*. I.Def.15

And in these *DA* equals *DB*, therefore the remainder *AL* equals the remainder *BG*. C.N.3

But *BC* was also proved equal to *BG*, therefore each of the straight lines *AL* and *BC* equals *BG*. And things which equal the same thing also equal one another, therefore *AL* also equals *BC*. C.N.1

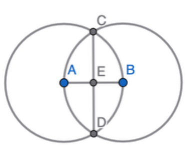
Therefore the straight line *AL* equal to the given straight line *BC* has been placed with one end at the given point *A*.

Q.E.F.

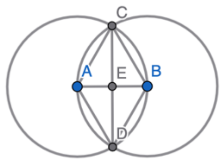
Image from: http://aleph0.clarku.edu/~djoyce/elements/bookI/propI2.html

Answers:

1.

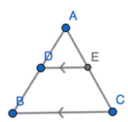


2. A) make a circle with radius AB and center A. Repeat at center B. This is allowed by postulate 3. B) Connect their intersections (C and D), this is allowed by postulate 1. The midpoint E is where $\overline{CD}$ intersects $\overline{AB}$.
3.

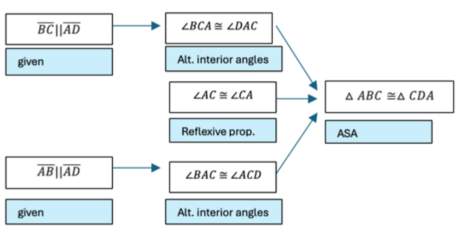


AB= AD = CB=DB, the are all radii of circles we constructed to be the same radius. CD= CD, by the reflexive property. Triangle CAD is congruent to triangle CBD by SSS. So angle ACE = angle BCE by CPCTC. Thus, triangles ACE and triangle BCE are congruent by SAS. And thus AE is congruent to BE by CPCTC.

4. Propositions must be proven, postulates are accepted or assumed to be true. We prove propositions must be true as a result of the postulates we accepted.
5. His first step in this proof was to connect to points (that's fine) but the he tells us to make and equilateral triangle of that distance. That was not a postulate so we needed to prove it could be done before we could accept this proof.
6. They could have started from different premises.
7. ASA
8. $\angle A \cong \angle F$
9. Draw a line DE parallel to BC and inside the original triangle. Angle A is shared, $\angle B \cong \angle D$ and $\angle C \cong \angle E$ as they are corresponding angles of a transversal.



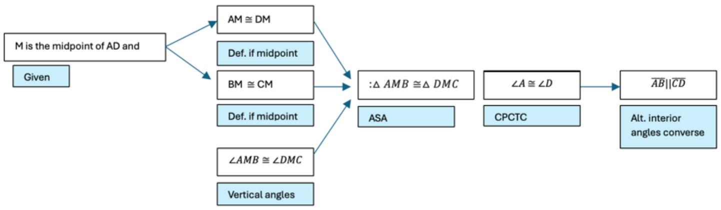
10.



11. $\overline{BC} \cong \overline{BC}, \angle DEB \cong \angle AEC$ and $\angle D \cong \angle A$
12. One other set of corresponding sides being congruent.
13.

Statement	Justification
BA ⊥ BE DE ⊥ BE	Given
∠B and ∠E are right angles	Definition of ⊥
∠B ≅ ∠E	Post. 4
C is the midpoint of BE	given
$\overline{BE} \cong \overline{EC}$	Def of midpoint
∠DCB ≅ ∠ECA	given
∠DCB=∠DCA + ∠ACB. And ∠ECA = ∠DCA + ∠DCE	Angle addition postulate (CN5)
∠DCA + ∠ACB ≅ ∠DCA + ∠DCE	substituion
∠ACB ≅ ∠DCE	C.N 3
ΔABC ≅ ΔDEC.	ASA

14.



15. Write your own proof but here is guidance. You will get triangle OCN is congruent to triangle to GCN by SSS. Then the two smaller angles formed at C are congruent by CPCTC. Now the two small triangles at the top of the kite are congruent by SAS. Call the intersection of the diagonals M. The two angles at M inside the small triangles are congruent by CPCTC, however they are also a linear pair. To be both congruent and a linear pair they must be right angles. Thus, the lines intersect at right angle and therefore are perpendicular.