Lesson 7.1 — Teacher Reference Notes

Ratios That Refuse to Change

Unit 7: Trigonometry • Day 1 of unit • 90 minutes: 45 min discovery + 45 min consolidation

At a Glance	
	Pacing: Discovery task (~45 min) • Gallery walk (~10 min) • Consolidation Part A: naming the ratios (~20 min) • Consolidation Part B: building the class trig table (~15 min) • Guided notes + CYU (~10 min or homework) **Materials:** Protractors, rulers, pencils, and paper for each group (not boards — precision matters) • Large class trig table displayed on wall (ready to be filled in) **Nav instrument rows active:** Row 1 (Trig Ratios) activates today — all three levels ⚠️ **Paper not boards:** This task is explicitly done on paper with pencil and ruler. Thick markers at boards introduce angle and length measurement error that obscures the constancy of the ratios. Lesson learned in Year 1. **SOH-CAH-TOA timing:** Introduce only after students have discovered the ratios. If introduced first, it becomes a formula to memorize rather than a name for something already understood. **45° group:** Make sure a Group is assigned 45°. These values are used in Lesson 7.4. If you have fewer than 7 groups, add this row to the class table manually.

OBJECTIVES

By the end of this lesson students can:

- Draw a family of similar right triangles anchored to a fixed acute angle and measure the three side ratios in each.
- State that these ratios are invariant across the family and depend only on the angle.
- Explain why the ratios are constant using AA similarity.
- Name the three ratios sine, cosine, and tangent, and connect each to SOH-CAH-TOA.
- Label opposite, adjacent, and hypotenuse relative to a given acute angle, and recognize that opposite and adjacent swap when the reference angle changes.
- Contribute values to the class trig table and read values from it.

The conceptual heart of this lesson — read before class

Students are not encountering ratio for the first time. They have used proportional reasoning since Unit 1, understood scale factor in Unit 6, and seen that similar triangles have proportional sides. The shift today is subtle but important: they move from comparing ratios between two triangles to recognizing that a ratio within a single triangle is a fixed property of its angle alone.

That shift is the conceptual birth of trigonometry. It means that if you know one acute angle, you know everything about the shape of the triangle — all three ratios are determined. The trig function is not a new idea. It is a name for a ratio the student has already found.

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GROUP ANGLE ASSIGNMENTS

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
15°	20°	30°	45°	50°	60°	80°

Distribute protractors, rulers, and pencils. Students work at tables on paper. Tell groups only their assigned angle and the instruction below.

THE DISCOVERY TASK (≈45 MIN)

Task prompt — read aloud or write on board

You have been assigned an acute angle. Draw a right triangle that contains your assigned angle. Then create two smaller versions of that same triangle and two larger versions. You should have a family of five right triangles — all the same shape, but different sizes. Look inside each triangle. Are there ratios between pairs of sides that stay the same across your whole family? Organize your results. When you have found something, ask: what determines this ratio?

⚠️ Precision note from Year 1

How close students' ratios are depends largely on how accurately they measured angles. The following approaches from Year 1 significantly improved precision:

- Groups who used the Pythagorean theorem to calculate the hypotenuse (rather than measuring it) got a constant ratio every single time — measurement error was eliminated from one side.
- One group drew one triangle, measured its sides carefully, then multiplied all sides by a constant scale factor to produce the other triangles. This guaranteed a constant ratio and avoided measurement error entirely. Consider seeding this approach if groups are struggling to see the constancy.
- Providing three sides and one angle, then asking students to make similar triangles, is a strong alternative framing that controls for angle measurement error.

Circulation questions

To push beyond the scale factor: if a group stalls at 'the scale factor stays the same', ask: 'That compares one triangle to another. What about relationships inside each triangle? Is there a ratio between two sides of the same triangle that stays the same?'

To establish the why: 'How do you know all your triangles are the same shape? Where is the AA similarity?'

To develop vocabulary: 'Can you name the sides relative to the angle you were given? Which side is across from the angle? Which is touching it but not the hypotenuse?' Let them find the language before you provide it.

Extension — give individually to groups that finish early

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Will these ratios be the same for every right triangle, or only for triangles with your assigned angle? Make a conjecture. Test it by comparing with another group. If I tell you one acute angle of a right triangle, do I need to tell you the other one? What determines all of the side ratios in a right triangle?

GALLERY WALK (≈10 MIN)

Groups examine each other's triangle families. Ask four questions in sequence:

- 'Within one group's family, what stays the same? What changes?'
- 'Across different groups' families, what changes? What determines the change?'
- 'Which group's ratios are most reliable? What did they do differently?'
- 'If someone gives you just one acute angle of a right triangle, what else is automatically determined?'

What to pull from the gallery walk into consolidation

Students should be arriving at: the ratios within a family are constant, ratios vary across families, and one acute angle determines all the ratios. If any group found a particularly clean result, name their approach explicitly.

CONSOLIDATION PART A — NAMING THE RATIOS (≈20 MIN)

Narrative Arc — The central message — say this exactly

"The ratio is constant depending on the angle. We just give it a name. Sine is the name of the ratio of the opposite side to the hypotenuse. Nothing more complicated than that — just the name of the ratio you found today."

Distribute guided notes. Introduce opposite, adjacent, and hypotenuse. Then present the three ratios and SOH-CAH-TOA.

sine (sin)	cosine (cos)	tangent (tan)
sin θ = opposite / hypotenuse **SOH**	**cos θ = adjacent / hypotenuse** **CAH**	**tan θ = opposite / adjacent** **TOA**

'Opposite' and 'adjacent' are always defined relative to a specific acute angle. They swap when you switch which angle you are referencing. The hypotenuse is always opposite the right angle — it never changes.

SOH-CAH-TOA timing note

SOH-CAH-TOA is a useful mnemonic. Introduce it only after students have discovered the ratios — never before. If introduced first it becomes a formula to memorize rather than a name for something already understood.

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The circleometry aside — worth one minute

For centuries, trigonometry was entirely about circles — the modern triangle-based presentation came later through 15th and 16th century European mathematicians. The word 'sine' is a Latin mistranslation of an Arabic translation of a Sanskrit word for 'half-chord.' Only in precalculus will students reconnect these ratios to the unit circle. Mention it briefly; it plants a seed.

Calculator note — tell students before Part B

Angles can be given in degrees or radians. Press MODE and choose Degrees. In this course we use degrees throughout. When computing trig values on a calculator, always verify the mode first.

CONSOLIDATION PART B — BUILDING THE CLASS TRIG TABLE (≈15 MIN)

Students return to their groups. Each group carefully computes sine, cosine, and tangent for both acute angles in their triangles and contributes values to the shared class trig table. Average values across the family of triangles to increase precision.

The complementary angle columns

The right column headers — sin (comp.), cos (comp.), tan (comp.) — record the trig values for the complement of the assigned angle. This will reveal the pattern $\sin(\theta) = \cos(90° - \theta)$ without any extra work. Name it after the table is complete: 'What do you notice about $\sin(\theta)$ and $\cos(90° - \theta)$ across every row?' This is the origin of the word cosine — complement's sine.

Class trig table — post on wall and leave up through Lesson 7.3:

Angle	sin	cos	tan	sin (comp.)	cos (comp.)	tan (comp.)
15°						
20°						
30°						
45°						
50°						
60°						
80°						

Patterns to prompt after the table is complete

- How does sine change as the angle increases from 15° to 80°?
- How does cosine change over the same range?
- What do you notice when you compare sin(30°) and cos(60°)? sin(20°) and cos(70°)?
- What is tan(45°)? Why does that make sense from the triangle?

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> *These observations foreshadow Lesson 7.3 (inverse trig) and Lesson 7.4 (special right triangles). Do not require students to memorize them now — pointing them out is sufficient.*

STUDENTS MAKE NOTES ON

Guided notes (not making-meaning notes) are appropriate for this lesson. Students do not yet have the correct vocabulary to generate their own summary.

> ✎ **Students make notes on:**
>
> - The big idea: one known acute angle in a right triangle guarantees similarity to every other right triangle with that angle (AA). In similar triangles, side ratios are constant — so trig ratios are properties of the angle alone.
> - Definitions: opposite, adjacent, hypotenuse relative to a specific acute angle
> - The three ratios: $\sin \theta$ = opp/hyp, $\cos \theta$ = adj/hyp, $\tan \theta$ = opp/adj — with SOH-CAH-TOA as mnemonic
> - A labelled right triangle diagram showing all three sides relative to one acute angle
> - The class trig table (students record values their group contributed; teacher fills in the rest)

> ▶ **Narrative Arc** — End of consolidation — transition to Lesson 7.2
>
> *"You just built a trig table from scratch using similar triangles. That table tells you the ratios for specific angles. Next lesson we ask the reverse question: if you know the ratios, can you find the sides? And if you know the sides, can you find the angle?"*

Connection Points	• Draws from Unit 6: AA similarity is the reason the ratios are constant — all right triangles with the same acute angle are similar • Draws from Unit 6: scale factor is what changes between triangles in a family; the trig ratio is what stays constant within each triangle • Draws from Unit 4: the triangle angle sum theorem is why knowing one acute angle determines the other ($90° + \theta$ + comp. $= 180°$) • Feeds into Lesson 7.2: the trig ratios established today are used immediately to solve triangles • Feeds into Lesson 7.3: inverse trig reverses the direction — from ratio to angle • Feeds into Lesson 7.4: the 45° and 30°/60° rows from today's table are the special right triangle values

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