Check Your Understanding 8.4

Show all work and reasoning. Spiral problems revisit earlier rows — use them to keep those ideas sharp.

Basic	Intermediate	Advanced
Row 1 Spiral — What is a circle? → *Defining the object*		
Basic — Spiral **1.** Circle C: $(x + 2)^2 + (y - 5)^2 = 49$. State the center and radius.	*Intermediate — Spiral* **2.** For circle C, is (5, 5) on, inside, or outside? Justify.	*Advanced — Spiral* **3.** Circle C has a horizontal diameter — give its two endpoints. Then write the equation of the concentric circle with twice the radius.
Row 2 Spiral — All circles are similar → *Why proportional reasoning works*		
Basic — Spiral **4.** A coin has radius 1 cm; a clock face has radius 15 cm. What is the similarity ratio mapping the coin onto the clock face?	*Intermediate — Spiral* **5.** The coin's circumference is 2π cm. Without calculating, state the clock face's circumference.	*Advanced — Spiral* **6.** A 120° sector of the coin has arc length $2\pi/3$. Explain two ways — similarity ratio, then the arc-length formula — why the clock's 120° sector arc is 10π, and what it means that both agree.
Row 3 Spiral — Arc length and sector area → *Applying proportional reasoning*		
Basic — Spiral **7.** A circle has radius 6; a sector has central angle 150°. Find the arc length.	*Intermediate — Spiral* **8.** For that sector, find the sector area.	*Advanced — Spiral* **9.** A second sector of the same circle has area equal to three times its arc length (numerically). Find the central angle, and explain what this says about arc length and sector area for this circle.
Central and inscribed angles → *The spine of the unit*		
Basic **10.** O is the center; arc AB = 110°. (a) What is the central angle AOB? (b) P is on the major arc — what is the inscribed angle APB? (c) Q is on the minor arc — what is the inscribed angle AQB?	*Intermediate* **11.** ABCD is inscribed in a circle with angle A = 73°. (a) Find angle C. (b) Angle B = 2x + 10 and angle D = 4x − 20 — find x and both angles. **12.** AB is a diameter; C is on the circle; angle CAB = 38°. (a) Find angle ACB and explain. (b) Find angle ABC. (c) Find the central angle AOC.	*Advanced* **13.** Two chords AB and CD intersect inside a circle at P; arc AC = 80° and arc BD = 60°. Find angle APC, justifying with the inscribed angle theorem. **14.** Prove: if two inscribed angles subtend the same arc, they are equal. Then explain in one sentence why this is obvious once you know the inscribed angle theorem.
Algebra Spiral — Exponent Rules		
Simplify. Write every answer with positive exponents. (a) $(12a^5b^2) \div (3a^2b)$ (b) $(x^{-2}y^3) \div (x^4y^{-1})$ (c) $(3m^2n)^3$ (d) $(2x^0)(5x^3)$		

Answers — CYU 8.4

Row 1 Spiral — What is a circle?

1. Center (−2, 5), radius 7.

2. Distance from (−2, 5) to (5, 5) is 7 = r, so the point is on the circle.

3. Endpoints (−9, 5) and (5, 5). Concentric circle (r = 14): $(x + 2)^2 + (y - 5)^2 = 196$.

Row 2 Spiral — All circles are similar

4. 15.

5. $15 \cdot 2\pi = 30\pi$ cm.

6. By ratio: $15 \cdot (2\pi/3) = 10\pi$. By formula: $(120/360) \cdot 2\pi(15) = 10\pi$. They must agree because the arc-length formula is itself a consequence of similarity.

Row 3 Spiral — Arc length and sector area

7. $(150/360) \cdot 2\pi(6) = 5\pi$.

8. $(150/360) \cdot \pi(36) = 15\pi$.

9. Area = $(\theta/360)(36\pi)$, arc = $(\theta/360)(12\pi)$; area = 3·arc reduces to 36 = 36 — true for every angle. For r = 6, sector area always equals 3 × arc length, since area = $(r/2) \cdot$arc.

Central and inscribed angles

10. (a) 110°. (b) 55° (half the central angle). (c) Q subtends the major arc (250°), so AQB = 125°. Note 55 + 125 = 180.

11. (a) 180 − 73 = 107°. (b) $(2x+10)+(4x-20) = 180 \rightarrow x = 95/3$; B = 220/3 ≈ 73.3°, D = 320/3 ≈ 106.7°.

12. (a) 90° — Thales' theorem (angle in a semicircle). (b) 180 − 90 − 38 = 52°. (c) Central angle AOC = 104°.

13. Angle APC = ½(arc AC + arc BD) = ½(80 + 60) = 70° — the intersecting-chords angle, which follows from the inscribed angle theorem.

14. Each inscribed angle equals ½(arc AB), so the two are equal. Once an inscribed angle is determined entirely by the arc it subtends, any two over the same arc must match.

Algebra Spiral — Exponent Rules

(a) $4a^3b$

(b) y^4/x^6

(c) $27m^6n^3$

(d) $10x^3$