

Unit 5

Quadrilaterals

7 lessons • CYU replaces homework throughout

Unit Navigation Instrument

Big Idea: Every quadrilateral proof is a triangle proof in disguise. The diagonal reveals the triangles, and the triangles reveal everything else.

Subtopic	Basic	Intermediate	Advanced
Classification and Definitions → Organizing the family	Name, define, and sketch each member of the quadrilateral family. Identify which properties a given shape must have based on its definition.	Determine whether a given quadrilateral belongs to a named category. Use properties to place it correctly in the hierarchy.	Explain why a property of a broader category automatically transfers to a subcategory — and give an example of a property that does not transfer in the other direction.
Parallelogram Properties → Proving the engine	State the four core parallelogram properties as conditional statements. Use them to find missing angles and side lengths.	Use parallelogram properties and their converses algebraically. Determine whether a quadrilateral with given measurements is a parallelogram.	Prove a parallelogram property using triangle congruence. Prove the converse that if both pairs of opposite sides are equal the quadrilateral is a parallelogram.
Special Parallelograms → Adding constraints, inheriting results	State the defining additional constraint for rectangles, rhombi, and squares. State which diagonal properties each has.	Use the diagonal properties of rectangles and rhombi to solve algebraic problems. Determine which type a special parallelogram is from given information.	Prove the diagonal property of a rectangle or rhombus. Explain why a square's diagonal properties follow without separate proof.
Trapezoids and Kites → Structure outside the parallelogram family	Define trapezoid, isosceles trapezoid, and kite. State the key angle and diagonal properties	Use the properties of isosceles trapezoids and kites to solve algebraic problems.	Prove a property of an isosceles trapezoid or kite using triangle congruence.

	of each. Use them to find missing measurements.	Verify a property using coordinates.	
Coordinate Methods → <i>A second proof system</i>	Use slope, distance, and midpoint to identify properties of a quadrilateral given its coordinates. State what each tool checks.	Given coordinates of a quadrilateral, determine which type it is by checking the necessary conditions. Explain which conditions are sufficient.	Write a complete coordinate proof that a quadrilateral with given vertices is a specific type. Justify every claim with a calculation.

The big idea is not a slogan — it is a genuine proof strategy. Every proof in this unit uses a diagonal to reveal triangles, and every conclusion follows from triangle congruence tools established in Units 3 and 4. The Advanced column on Classification captures the Unit 2 conditional logic thread: properties of a set transfer to subsets, but not in the other direction. A teacher who is behind schedule may treat the properties as accepted facts; the proofs are good practice but not load-bearing for what comes after. The one proof worth preserving under time pressure is the parallelogram diagonal proof: it is the clearest single demonstration of the hidden triangle strategy and the one students are most likely to encounter on standardized assessments.

What This Unit Is About

Every quadrilateral proof in this unit is a triangle proof in disguise. Draw a diagonal, reveal the triangles, and the properties follow from the congruence tools students built in Units 3 and 4. That idea — named explicitly as the unit’s big idea from the first lesson — is not a slogan. It is a genuine proof strategy that works for every shape in the unit without exception.

The unit opens with the Birthday Cake Problem, a constraint-based task drawn from Peter Winkler’s *Mathematical Mind-Benders* (A K Peters, 2007) and popularized by Alex Bellos in *The Guardian*, October 21, 2019. Students find all configurations of four points with at most two distinct pairwise distances — and discover quadrilateral shapes as solutions to a problem rather than as objects to memorize. The bucket sort task that follows asks students to sort shapes at the boards into self-defined categories. The hierarchy emerges from the buckets, not from a lecture. Formal definitions come in consolidation, after students have earned the intuition that justifies them.

From there the unit builds in a deliberate logical sequence. Parallelogram properties are established first because every rectangle, rhombus, and square is a parallelogram — proving something for the general case automatically covers all the special cases. The special parallelogram lessons add one constraint at a time and prove the resulting

diagonal properties: equal diagonals for rectangles via SAS, perpendicular diagonals for rhombuses via the base angles theorem from Unit 4. The base angles theorem connection is made explicit — students are not learning a new technique but applying an existing tool in a new context. The square properties then follow without new proof, purely by logical inheritance from both parent shapes. This is the clearest illustration in the course of why the hierarchy matters: the most constrained shape requires the least new work.

Trapezoids and kites sit outside the parallelogram family and have their own proof structure. Both still follow the hidden triangle strategy — the auxiliary constructions reveal triangles, and congruence does the work. Symmetry is present in both shapes and worth naming as an observation that helps students anticipate where the equal parts will be, but the proofs themselves are triangle congruence arguments.

The unit closes with a coordinate proof day that revisits every property established through Euclidean methods and re-proves them using slope, distance, and midpoint. The closing discussion makes the axiomatic point explicit: Euclidean geometry and coordinate geometry are different formal systems built on different foundations, yet they agree on every result in this unit. Students who have been with the course since Unit 2's axioms lesson will recognize the structure — different starting premises, same logical conclusions.

The connections to other units are direct. The hierarchy and property-transfer logic draws on Unit 2's work on conditionals, converses, and Venn diagrams. The parallelogram proofs use the alternate interior angles theorem and converse from Unit 3. The rhombus diagonal proof invokes the base angles theorem from Unit 4. The coordinate methods draw on slope and distance from Unit 1. The two-systems closing discussion plants a seed that Unit 8 will harvest when students prove circle properties algebraically.

What Happened When This Was Taught as Facts

Unit 5 was taught traditionally in Year 1 — properties presented directly, demonstrated by the teacher, practiced through exercises. It worked in the narrow sense: students could identify shapes, apply formulas, and demonstrate proficiency on assessments. But something was missing, and it was noticeable.

Students seemed disengaged in a way that had not been true in earlier units. The material felt to them like a set of arbitrary facts to remember and demonstrate proficiency with on a quiz — nothing more. We still tried to show them how the properties were built from everything we had learned in the course, and how new results could be built from those properties. But it just did not seem to land the same way with the more traditional approach.

The contrast was instructive. In Units 3 and 4, students had proven theorems and felt the weight of what that meant — a result that must be true, that follows necessarily from what came before, that can be cited in future arguments without re-proof. In Unit 5, taught as facts, the results were just things to know. The logical structure that makes the hierarchy interesting — properties flowing downward through subsets, the square needing no new proof because it inherits everything — was stated but did not seem to register.

The redesigned unit attempts to restore what was missing. The Birthday Cake Problem and the bucket sort are not decorative. They are the experiences that give students the intuition from which the formal definitions earn their meaning. The proofs are not supplementary exercises. They are the moments when students discover that the hierarchy is not a memorization scheme but a logical system — and that the work they did in Units 3 and 4 is what makes it provable.

This is a new design. It has not been tested in the classroom in this form. Teachers who use it should expect to iterate and are warmly invited to bring their notes back to improve future versions of this chapter.

Lesson Sequence Overview

Lesson 5.1: The Birthday Cake Problem and the Quadrilateral Family — constraint task, bucket sort, hierarchy, Unit 2 conditional logic connection

Lesson 5.2: Parallelogram Properties — four properties discovered and proven, hidden triangle strategy named, converses established

Lesson 5.3: Special Parallelograms — rectangle diagonal theorem, rhombus diagonal theorem via base angles, square by inheritance

Lesson 5.4: Trapezoids and Kites — isosceles trapezoid properties via HL, kite perpendicular diagonals via SSS, symmetry as argument

Lesson 5.5: Coordinate Proof Day — slope, distance, midpoint, three-tier board work, two-systems closing discussion

Lesson 5.6: Practice Day

Lesson 5.7: Assessment